POTENTIAL THEORY AND DEGENERATE PARTIAL DIFFERENTIAL OPERATORS

Edited by

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TABLE OF CONTENTS

Foreword

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The study of degenerate elliptic operators has followed till recent years two different lines.

The first one, beginning with the pionering work of Hörmander, relies on the property of the solutions to the properties of the *symbol* of the operator under consideration. The methods used here are typical of linear Analysis and allow us to study the *global* regularity of the solutions in the case of operators with smooth coefficient (we emphasize that *local* regularity results cannot, in general, be proved by these methods).

The second one starts with the work of E. Fabes, D. Jerison, C. Kenig and R. Serapioni on weighted uniformly elliptic operators where the weight is in the A_2 Muckenhoupt class. The goal of the theory is an extension of the De Giorgi–Moser–Stampacchia theory for uniformly elliptic operators with discontinuous coefficients, that is essentially a *local* theory. In this framework some problems remain open; we mention in particular two of them : (1) find global conditions on the weight allowing to develop a De Giorgi–Moser–Stampacchia theory; (2) the study of the local regularity for general nonlinear elliptic (or parabolic) weighted problems (some results are known in the case of quasilinear problems also with "natural" growth conditions).

In the eighties a fundamental step in a local theory for subelliptic operators has been the definition by C. Fefferman and A. Nagel, E. Stein, S. Weinger of a distance related to a subelliptic operator (in the sense of Hörmander). After this result many authors have proved local results for same subelliptic operators on R^N or on a manifold but no general theory was constructed. In the sixties Dirichlet forms have been introduced by Deny and I will mention as a good introduction the CIME lecture given by Deny in 1968. Dirichlet forms have rapidly become a cross-point between Potential theory, the studies on linear semigroups and on stochastic processes. Only in the last years Dirichlet forms have been recognized to be a useful tool to unify and generalize the previous results on weighted and subelliptic operators giving a general De Giorgi–Moser–Stampacchia theory, that is founded only on a duplication property for the intrinsic balls with respect to the measure on the space under consideration and on a scaled Poincaré inequality. The interest was so also attracted by the representations of Dirichlet forms in a local

and in a nonlocal part, by the Potential theory for Dirichlet forms as well as by the generalization of Dirichlet forms to infinite dimensional spaces

The Conference has been organized to compare the recent results and point of view on the above topics.

I will at first thank the lecturers (in particular those who have sent their manuscript for the proceedings of the Conference) for the deeply interesting contributions and all the people present at the Conference for the stimulating discussions.

I thank also the CNR, the CNR Research Project "Problemi variazionali irregolari", the MURST Research Project "Calcolo delle Variazioni, Pb. unilateri e di frontiera libera etc." for financial support.

The Department of Mathematics of Parma University has kindly given hospitality to the Conference, in particular Prof. Silvana Marchi has taken care of the local organization in Parma and without her the Conference should be impossible.

Finally I am obliged to Kluwer Academic Publishers and to "Potential Analysis" for the publication of the proceedings of the Conference.