Three-Dimensional Contact Problems

SOLID MECHANICS AND ITS APPLICATIONS
Volume 93

Aims and Scope of the Series

The fundamental questions arising in mechanics are: *Why?*, *How?*, and *How much?* The aim of this series is to provide lucid accounts written by authoritative researchers giving vision and insight in answering these questions on the subject of mechanics as it relates to solids.

The scope of the series covers the entire spectrum of solid mechanics. Thus it includes the foundation of mechanics; variational formulations; computational mechanics; statics, kinematics and dynamics of rigid and elastic bodies: vibrations of solids and structures; dynamical systems and chaos; the theories of elasticity, plasticity and viscoelasticity; composite materials; rods, beams, shells and membranes; structural control and stability; soils, rocks and geomechanics; fracture; tribology; experimental mechanics; biomechanics and machine design.

The median level of presentation is the first year graduate student. Some texts are monographs defining the current state of the field; others are accessible to final year undergraduates; but essentially the emphasis is on readability and clarity.

For a list of related mechanics titles, see final pages.

Three-Dimensional Contact Problems

by

V.M. ALEXANDROV

Moscow State University,
Department of Mechanics and Mathematics,
Moscow, Russia

and

D.A. POZHARSKII

Rostov-on-Don State University,
Mechanics and Applied Mathematics Institute,
Rostov-on-Don, Russia

KLUWER ACADEMIC PUBLISHERS
DORDRECHT / BOSTON / LONDON

A C.I.P. Catalogue record for this book is available from the Library of Congress.

ISBN-13: 978-1-4020-0387-5 e-ISBN-13: 978-94-010-9893-9
DOI: 10.1007/978-94-010-9893-9

Published by Kluwer Academic Publishers,
P.O. Box 17, 3300 AA Dordrecht, The Netherlands.

Sold and distributed in North, Central and South America
by Kluwer Academic Publishers,
101 Philip Drive, Norwell, MA 02061, U.S.A.

In all other countries, sold and distributed
by Kluwer Academic Publishers,
P.O. Box 322, 3300 AH Dordrecht, The Netherlands.

Printed on acid-free paper

Contents

Preface **xi**

Chapter 1. Contact problems for a half-space **1**

1.1. Introduction 1
1.2. Elasticity theory 2
1.3. Frictionless contact problems for a half-space 4
1.3.1. The formula for the normal displacement 4
1.3.2. A strip punch on a half-space 8
1.3.3. General formulation of the contact problem 12
1.4. A circular punch on a half-space 14
1.4.1. The method of paired integral equations 14
1.4.2. Special cases 16
1.5. An elliptic punch on a half-space 18
1.5.1. The solution for a polynomial base of the punch . . . 19
1.5.2. The solution for an elliptic paraboloid 23
1.5.3. Two identical elliptic punches 27
1.5.4. An elliptic punch having a circular hole 30

Chapter 2. Contact problems for layers and half-spaces **33**

2.1. Equilibrium under given normal loads 33
2.1.1. General solution 33
2.1.2. Boundary conditions 35
2.1.3. Formulation of the contact problem 38
2.1.4. Properties of the function $L(u)$ 39
2.2. Integral equations for layer contact problems 41
2.2.1. Basic parameters and general results 41
2.2.2. Integral characteristics of the solution 44
2.2.3. Estimates for the force and moments 46
2.3. A circular punch on a layer 48

2.3.1. Integral equations 48
2.3.2. Properties of the kernel for axial symmetry 51
2.3.3. The asymptotic solution for large λ 53
2.3.4. The asymptotic solution for small λ 56
2.3.5. Another asymptotic solution for small λ 58
2.3.6. The method of orthogonal polynomials 62
2.4. A strip punch on a layer 63
2.4.1. One-dimensional integral equation 63
2.4.2. General results 65
2.4.3. Krein's method 67
2.4.4. Asymptotic solutions 69
2.4.5. The method of orthogonal functions 71
2.4.6. An annular punch on a layer 72
2.5. Contact problems for a thick layer 74
2.5.1. The asymptotic solution 74
2.5.2. An elliptic planar sloping punch 76
2.5.3. The force and moments 79
2.6. Contact problems for a thin layer 80
2.6.1. The confluent solution 80
2.6.2. The boundary layer solution 84
2.6.3. An important special case 85
2.6.4. Examples 87
2.6.5. Determination of the force 89
2.7. A circular punch on a layer (examples) 91
2.7.1. A planar punch 91
2.7.2. A parabolic punch 93
2.7.3. A planar sloping punch 95
2.8. A narrow rectangular punch on a half-space 97
2.8.1. Basic assumptions 97
2.8.2. The integral equation 99
2.8.3. Perturbation of the kernel 100
2.8.4. A planar punch 101
2.8.5. Numerical results 103
2.9. Galanov–Newton method 105
2.9.1. Nonlinear boundary equations 105
2.9.2. Testing the computer program 110
2.9.3. Contact problem for a two-layer foundation 113

Chapter 3. Contact problems for a cylinder **119**
3.1. Equilibrium under given normal loads 119

3.1.1. An infinite cylinder 119
3.1.2. A finite cylinder under special conditions 123
3.1.3. A layer under special conditions 126
3.1.4. Homogeneous solutions 128
3.2. Contact problems for a cylinder, and a space with a cylindrical cavity 130
3.2.1. Integral equations 130
3.2.2. Important singular integrals 135
3.2.3. Solution for a thick cylinder 136
3.2.4. Solution for a thin cylinder 139
3.3. Interaction between a cylinder and deformable sleeves 145
3.3.1. Formulation and asymptotic solutions 145
3.3.2. Two identical sleeves 151
3.3.3. The method of orthogonal polynomials 160
3.4. Periodic system of sleeves on a cylinder 163
3.4.1. Formulation and reduction to infinite systems 163
3.4.2. Reduction to singular integral equations 167
3.4.3. Rigid sleeves 177
3.5. Interaction between a cylinder and a wheel or cylindrical shell 181
3.5.1. Interaction between a cylinder and a wheel 181
3.5.2. Interaction between a cylinder and a shell 185
3.6. Contact problems for a circular plate 191
3.6.1. Formulation and reduction to an infinite system . . . 191
3.6.2. The method of a superposition 197
3.6.3. The plate with a noncylindrical lateral surface 202
3.7. Contact problems for a finite cylinder 207
3.7.1. Homogeneous solutions 207
3.7.2. Contact problems 208

Chapter 4. Contact problems for a wedge 215
4.1. Equilibrium under given normal loads 217
4.1.1. One face of the wedge is stress-free 218
4.1.2. Other types of boundary conditions 222
4.1.3. Analysis of the Fredholm integral equations 226
4.1.4. Example: problem A for a quarter-space 230
4.1.5. Inversion of integral operators 233
4.1.6. Papkovich–Neuber functions 235
4.2. Strip punch on a wedge face 239
4.2.1. Reduction to infinite systems 230
4.2.2. Method of dual integral equations 244

4.3. Wedge-shaped punch on a wedge face 247
4.3.1. Asymptotic method 248
4.3.2. Bubnov–Galerkin method 255
4.4. Elliptic punch on a wedge face 261
4.5. Contact problem for a wedge with unknown contact area . . 271

Chapter 5. Contact problems for a cone 281
5.1. Equilibrium under given normal loads 282
5.2. Rigid or deformable sleeve on a cone 288
5.2.1. Integral equations 288
5.2.2. Solution for large λ 291
5.2.3. Solution for small λ 294
5.2.4. Deformable sleeve 297
5.3. Plane sloping annular punch on a half-space 300
5.3.1. Solution for a narrow annular punch 302
5.3.2. Solution for a wide annular punch 305
5.3.3. Another solution for a wide annular punch 307
5.4. Wedge-shaped punch on a cone 310
5.4.1. Asymptotic solution 312
5.4.2. Bubnov–Galerkin method 315
5.5. Contact problem for a cone with unknown contact area . . . 317
5.6. Contact problems for a spherical layer 323
5.6.1. A spherical layer 323
5.6.2. A spherical bearing 326
5.6.3. Sector of a spherical layer 330
5.6.4. Superposition of homogeneous solutions 337

Chapter 6. Contact problems for spherical lens 339
6.1. Equilibrium under given normal loads 341
6.1.1. Ceneral scheme for the 3-D case 341
6.1.2. An axisymmetric case 346
6.2. A truncated sphere 352
6.2.1. The method of dual equations 353
6.2.2. A non-axially symmetric case 357
6.3. A half-space with spherical hollow or jut 359
6.4. A space with spherical cavity 364
6.4.1. Solution for one punch 364
6.4.2. Two identical punches 373
6.5. Sphere, spherical hinge 375
6.5.1. A sphere 375

6.5.2. A spherical hinge 378

Appendix. Galanov's computer program **383**

Abbreviations for journals **389**

References **391**

Index **399**

Preface

The mechanics of contact between deformable bodies, so-called contact mechanics, is an important and actively developing part of continuum mechanics. Realistic boundary conditions on a surface of deformable body may be formulated only as a result of a solution of a contact problem. Therefore, the determination of the stress–strain state of a body is also connected with a contact problem.

The characteristic feature of contact problems is that they have mixed boundary conditions and may thus be reduced to integral equations. This work focuses on such integral equations, properties of their kernels, suggesting a large number of beautiful asymptotic and numerical solutions.

In this monograph a systematic treatment is given for analytical and numerical methods and results for many nonclassical three-dimensional contact problems in the linear theory of elasticity. We consider semiinfinite elastic bodies, such as half-spaces, layers, holes, cylinders, spaces with cylindrical or spherical cavities, wedges, cones, half-spaces with spherical holes; and finite elastic bodies, such as circular plates, finite cylinders, spheres and spherical segments.

The monograph includes a vast amount of material accumulated during scores of years in numerous publications by the authors and their colleagues.

Methods introduced in the book can also be applied to crack and inclusion problems in fracture mechanics as well as to other problems in the disciplines of mechanics, applied mathematics, mathematical physics and engineering with mixed boundary conditions.

For formulae, figures and tables, we use double enumeration where the first number is that for the chapter. For special mathematical functions, functional spaces and operators, we usually use gothic script.

We wish to thank Prof. G.M.L. Gladwell (University of Waterloo, Canada) for his careful editing the book, and Prof. W.L. Wendland (University of Stuttgart, Germany) for his useful remarks.

Our work was supported by the Alexander von Humboldt Foundation (Germany) and Russian Foundation for Basic Research (99-15-96012).

V.M. Alexandrov and D.A. Pozharskii
Moscow, Stuttgart, 2001