

THE LOGIC OF TIME

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THE LOGIC OF TIME

*A Model-Theoretic Investigation into the Varieties of
Temporal Ontology and Temporal Discourse*



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PREFACE

That philosophical themes could be studied in an exact manner by logical means was a delightful discovery to make. Until then, the only outlet for a philosophical interest known to me was the production of poetry or essays. These means of expression remain inconclusive, however, with a tendency towards profuseness. The logical discipline provides some intellectual backbone, without excluding the literary modes.

A master's thesis by *Erik Krabbe* introduced me to the subject of tense logic. The doctoral dissertation of *Paul Needham* awakened me (as so many others) from my dogmatic slumbers concerning the latter's monopoly on the logical study of Time. Finally, a set of lecture notes by *Frank Veltman* showed me how classical model theory is just as relevant to that study as more exotic intensional techniques.

Of the authors whose work inspired me most, I would mention *Arthur Prior*, for his irresistible blend of logic and philosophy, *Krister Segerberg*, for his technical opening up of a systematic theory, and *Hans Kamp*, for his mastery of all these things at once.

Many colleagues have made helpful comments on the two previous versions of this text. I would like to thank especially my students *Ed Brinksma*, *Jan van Eyck* and *Wilfried Meyer-Viol* for their logical and cultural criticism. The drawings were contributed by the versatile *Bauke Mulder*.

Finally, Professor *Hintikka's* kind appreciation provided the stimulus to write this book.

INTRODUCTION

Philosophers have had much to say about the nature of Time. Mathematicians and physicists add a lot more from their perspective. More recently, linguists are also becoming interested in the temporal constructions of natural language. Can a logician add anything of value to all this wisdom? In this book it will be shown which types of question come to the fore from a logical point of view.

One typical perspective in logic is the dual attention to matters of language (temporal discourse, in the present case) and matters of ontology (temporal structures). Thus, logic forms a bridge between linguistics and mathematics. This perspective is reflected in the organization of the book: temporal ontology occupies Part I, temporal discourse is treated in Part II. Another characteristic feature of modern logic is its multiplicity as to languages, theories of inference and formal semantics. This is just what is needed in the study of Time, as our intuitions on this subject are so diverse that no unique 'logical structure' is to be expected. Probably the most fundamental issue is that of discreteness versus continuity of Time. These complementary points of view permeate all of our thinking on the matter, and it is to be doubted if ever one can be definitively reduced to the other. Accordingly, this opposition forms the second main theme of this book, occurring in both parts. Finally, this multiplicity of logic does not undermine its methodological unity, as logicians are forever comparing the various approaches. This third tendency shows up in the book as well: the two complementary views of Time turn out to be related in several interesting ways.

For the purposes of our logical investigation, Time will be represented as a formal structure in the model-theoretic sense: a totality of temporal 'individuals' arranged in a temporal 'order'. Such static pictures are perfectly appropriate to the study of Space; but in the case of Time there remains an intuitive residue which is not captured by logical analysis. Our experience that 'time passes' transcends all theoretical description. A colleague once told me that, therefore, the present enterprise is a priori doomed to failure. Did not Heidegger show convincingly that the structure of Time can never be separated from that of our living experience? Quite

the opposite is true. Logical studies of Time do manage to isolate and investigate formal structures and, in doing so, our experience of this phenomenon is enhanced rather than dulled. “This thing all things devours”, as Tolkien makes Gollum say when “the time has come to ask something hard and horrible”. I find it alarming that time passes, and writing this book has even added one more tinge of melancholy to that feeling: the eternal logical structure is only there to see, not to be lived in.

Global objections like the previous one – which are just variations upon the familiar, tearful opposition of ‘logic versus life’ – need not be taken very seriously. But there may be a rational core to the complaint in the sense that it expresses the alienation between common sense and scientific views of Time. One of the aims of this study is precisely to help bridge this deplorable cultural gap by showing how scientific theories of Time may arise starting from common sense notions, such as ‘period’ or ‘event’.

Apparently a more debatable presupposition of this logical approach is that even a continuum of Time is represented as a point set of (‘discrete’) individuals connected by (‘external’) relations. Nevertheless, in reality, this analytical decomposition still allows for the most diverse explications of our vague intuitive notions of continuity and discreteness. Indeed, as so often in philosophy, one has to be an analyst the more to appreciate the whole. Thus, the ontological bias on this score seems to be rather harmless.

Scientists may feel, however, that it is too late for questions like these. Physics proceeds straight on to integer or real time; so, why fight rear-guard actions? Knock-down objections of this sort fail to impress me. Philosophy has been described as ‘the science of lost causes’, a banner that I would gladly follow. The importance of philosophy (and logic) consists to a large extent in the ‘conceptual sanctuary’ it provides, sheltering our cultural heritage from the vicissitudes of scientific (or political) fashion. And anyway, one should think more about seemingly irrevocable scientific choices, rather than less.

Before continuing these general considerations, the contents of this book will now be described in a little more detail.

Part I opens with a chapter in which various choices for temporal individuals and their relations are surveyed. A broad opposition emerges between a ‘point’ ontology on the one hand, and an ontology of extended ‘periods’ on the other. The former ontology is investigated systematically in Chapter I.2. First of all, which conditions are to be satisfied if a point structure is to qualify as a representation of Time? (A certain dissatis-

faction with the casual way in which most tense-logicians present their postulates, hurrying on to the 'respectable' structures of integer, rational or real time, motivated this part of the present research.) Direct axioms are derived, in ascending order of logical complexity, from the familiar idea of a 'river' of Time (Section I.2.1.). Several complete logical theories emerge (first-order or higher-order) of discrete, dense or continuous point structures. As it happens, such theories usually possess many 'non-standard' models as well, which are not expected in the first place. Temporal pathologies? Some order is created here by considering more global intuitions on the type of structure which is suitable for modelling Time, for instance 'homogeneity' or 'isotropy' (Section I.2.2.). These turn out to exclude many non-standard models; of the remaining ones, one gradually grows fond... Such temporal structures do not represent isolated choices: they are themselves related in several ways, and this categorial perspective is explored as well (Section I.2.3.).

The period ontology is developed in Chapter I.3. along these same lines. Only this time postulates are not extracted from pure intuition alone. Two standard 'interval structures' are presented, one discrete, the other continuous, to aid our imagination. Their complete theories are determined, again admitting of various non-standard models. As these two examples are derived from point structures, there is some poetic justice to this chapter: the point ontology is used to set up its rival. This is only one instance of the various connections existing between the two ontologies which are explored systematically in Chapter I.4. A beautiful harmony then emerges. On the one hand, point structures induce period structures through the formation of intervals (Section I.4.1.); on the other hand, point structures may be re-created from period structures by a limiting construction (Section I.4.2.). More specifically, points may be introduced as converging filters of periods; a mathematical idea with a philosophical pedigree, developed already by the metaphysician Alfred Whitehead, more than sixty years ago. This double connection relates the two categories in an enlightening way (Section I.4.3.). Notably, it allows one to have the best of both worlds, both philosophically and mathematically.

The final chapter of the first part is devoted to a yet different starting point, viz. that of event structures (I.5.). The classical road in tense logic would proceed from 'points' via 'periods' to 'events' – an order which is now reversed. There are analogies here with the so-called 'Causal Theory' of Space-Time, which is discussed briefly. All the while, the attempt is made to motivate formal conditions on structures at one level through

their construction from those at the previous level. In this way, an amazing amount of logical structure may be derived starting from the simple picture of a child arranging his toys around him.

In Part II the emphasis shifts to temporal languages. (The difference between the two parts is one in emphasis – an absolute separation between ‘ontology’ and ‘language’ being impossible.) A preliminary discussion of temporal discourse takes place in Chapter II.1., delineating the formal languages to be studied. The full wealth of natural language will not be tackled: there are enough problems as it is with extremely simple logically regimented sublanguages. One of these is Priorean propositional logic, presented in Chapter II.2. A general survey is given, distinguishing elementary model theory (Section II.2.1.), ‘correspondence theory’ connecting tense logic with the ‘classical logic’ of Part I (Section II.2.2.), as well as the ubiquitous ‘completeness theory’ of contemporary research (Section II.2.3.). Thus, also a new perspective is gained upon the topic of Chapter I.2.: the two parts of this book work in tandem.

Many researchers have advocated an ‘interval tense logic’ for various philosophical and linguistical reasons. Whether this revival of period discourse is a mere modern fashion cannot yet be judged at this stage. The present author found these ideas impressing themselves upon him in 1978, without any external pressure, social or otherwise. (But maybe, that is the tragic self-deception of all followers of fashion: to-day they *believe* in it!) This type of tense logic is developed in Chapter II.3., on the pattern of the previous chapter. Notably, the two standard ‘interval structures’ of Chapter I.3. are investigated in this light. Connections between these two varieties of tense logic occupy Chapter I.4. – as will be predictable by now from the symmetry of the book. A new type of ‘generic’ representation for period structures emerges in the process.

Finally, although this book has been devoted to Time only, its approach may be used equally well to study Space or Space-Time. Some indication of this is given in the, rather compact, Appendix A.

Clearly, there are limitations to the present subject matter, even given the restriction to logic. To mention three themes which are conspicuous by their absence: there is no *predicate tense logic*, the interplay between *tense and modality* occurs only marginally, and – maybe most importantly – our study does not cover the role of *temporality* in the context of formal theories. These would be obvious follow-up projects, now that the basic ground-work has been laid. More generally, this is a study in philosophical logic, with some of its virtues, but also many of its vices. Especially,

connections with current work in *natural science* or *empirical linguistics* are absent. This gap will have to be bridged eventually, if, for instance, the period paradigm is to become a healthy intellectual enterprise.

Limitations such as the above are unavoidable, and necessary. Indeed, I have found it hard to resist the temptation to abuse these virgin pages for voicing my opinions about *everything*. (The reader will notice various lapses throughout the book in discussions and notes.)

Having presented the subject matter, such as it is, it remains to say something about the method. First, the *logic* used in this book is *elementary*; and (hence) no technical depth is claimed for the theory developed here. Readers with a modest background in logical model theory should be able to follow the exposition throughout, especially Part I. Not exactly indispensable, but rather helpful for the appreciation of Part II, is a little knowledge of tense logic. In fact, this book is not intended as a competitor for existing (or forthcoming) introductory textbooks in that field.

Next, the *philosophy* occurs in this book in the form of numerous ‘discussions’: no coherent system is developed. Here are a few samples. Temporal intuitions of a Great Philosopher are scrutinized in a discussion of Kant’s ‘First’ and ‘Second Antinomy’ (Section I.2.1.; A): the outcome is rather disappointing. This is different with McTaggart’s ‘A-series’ and ‘B-series’ conceptions of Time (Section I.1.2.; Chapter I.5). Incidentally, I claim that the two are inter-reducible, as against McTaggart himself and Russell, who defended only (opposing) halves of this assertion. Moreover, inevitably, I had to try and throw some new light upon ‘Zeno’s Paradoxes’ – against the background of the point/period distinction (Chapter I.5.). Briefly, my claim here is that these may be detonated, so as to become (respectable) constraints upon any ‘punctual’ representation of period structures. These discussions are invariably *short*: I hate those sequences of page after boring page of what reviewers call ‘carefully spelt-out argument’. Indeed, the ratio of prose to ideas in many philosophical books is often an insuperable barrier to the logical reader. Moreover, the fact that these discussions occur rather casually is a conscious reflection of a discursive view of philosophy. It is a spice, only to be consumed in small doses, and never without a substantial meal of a different origin.

Another philosophical feature, or non-feature, is the *lack of drama*. Some readers of previous versions have complained that the story lacked ‘philosophical tension’; something which could only be created, it seems, by defending the position that the period approach is *better* than its classical rival (or vice versa). The dispassionate study of the merits of both

has even been branded as purely ‘technical’. Well, in that sense, the latter epithet loses its pejorative connotation. Indeed, when matters are presented like this, one could equally well define a ‘real philosopher’ as someone who cannot even study a purely intellectual subject without taking sides. (As the psychologist Jaap van Heerden once remarked, an ordinary person merely observes that she is different from other people (by definition, so to speak): a philosopher starts crying about the ontological chasm separating him from ‘The Other’.) Academic pin-pricks aside, if pressed for a verdict, I should say that no choice *ought to be* made. The period approach has the philosophical advantage, and it turns out to be technically viable – but, the complexities of its development are themselves an illustration of the technical advantage of its point competitor. Thus, we should want to keep what is good in both, and that is precisely what this book shows we can do.

A last point of philosophical method concerns the source of the ideas concerning Time to be found in this book. There is a lot of talk about ‘intuitions’ in contemporary logic, a tendency to which I have succumbed occasionally. (Ironically, so-called ‘exact philosophy’ is one of the last intellectual hide-outs of this nebulous talk.) What is never claimed, however, is any mysterious support for these intuitions: all the trees in this book are to be judged by their fruits.

Finally, in view of the many things which this treatise is *not*, one might think it was intended for a singleton audience. That is emphatically not the case: I sincerely hope that the themes and questions raised here will inspire many readers. Indeed, as there are more questions being raised in this book than answered, both technically and philosophically, the need for a co-operative audience to carry on the enterprise is rather obvious.

One point remains to be explained concerning the medium of this message. The unique language of this text is the result of converting Dutch thoughts into English phrases.