Proceedings of the
Ninth International Congress on
Mathematical Education

Proceedings of the Ninth International Congress on Mathematical Education

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Preface

With the attached CD, this book constitutes the Proceedings of the 9th International Congress on Mathematical Education (ICME 9), which was held in Tokyo/Makuhari, Japan, from July 31 to August 6, 2000. There were 2,012 registered participants from 70 countries, who were joined by 239 accompanying persons.

The previous sequence of these quadrennial ICMEs had taken place in Lyon (France), Exeter (United Kingdom), Karlsruhe (Germany), Berkeley (United States of America), Adelaide (Australia), Budapest (Hungary), Quebec (Canada) and Seville (Spain). Thus ICME 9 is the first ICME that was hosted by an Asian country.

The ICMEs are held under the auspices of the International Commission on Mathematical Instruction (ICMI), and on the occasion of ICME 9 the official hosts were Science Council of Japan and Japan Society of Mathematical Education. To execute actual jobs of organization, the official sponsors set up a National Organizing Committee (NOC) for ICME 9 in cooperation with Mathematics Education Society and Association of Mathematical Instruction, and with scientific assistance rendered by Mathematical Society of Japan, Japan Society for Industrial and Applied Mathematics, and other academic societies related with mathematics education. During the four years of preparation, members of the NOC and those of its subcommittees worked with devotion and feeling of honor to be involved in organization of this exceptional event in mathematical education, ICME 9.

As on previous ICMEs, the Executive Committee of ICMI set up an International Program Committee (IPC) for ICME 9 who was responsible for making the major scientific planning of the Congress. In positive appreciation of the main trends and achievements of recent ICMEs, the IPC decided that the general pattern used at Seville would be followed, while some innovative amendments and new measures were to be introduced, as mentioned by the IPC Chair in his greeting at the opening ceremony. For instance, Working Groups and Topic Groups kept their important positions in the program, while they were re-defined and re-named as Working Groups for Action and Topic Study Groups. There were 13 WGA and 23 TSG. As in recent ICMEs, the International Round Table was a plenary activity but at this time it was realized with distance speakers.

The IPC also invited four distinguished plenary speakers, fifty-two speakers to present regular lectures of 45-minutes on a large range of issues and matters, four national presentations (France, Italy, Japan, Malaysia) and reports from current ICMI studies, and from ICMI Affiliated Study Groups (PME, HPM, IOWME, WFNMC). As for organization of each of WGA or TSG, a Chief Organizer (CO: in many cases, two equivalent Co-COs), two or three Associated Organizers (AO) and a Local Assistant Organizer (LAO) were invited to form the Organizing Team of the

group. Among the invited special meetings, there was Forum of All Chinese Math Educators to celebrate the first coming of ICME to Asia.

As a whole, the IPC structured a rich and extensive scientific program to keep traditional value of meeting in face-to-face groups and of exchanging through individual contacts, and thus intended to encourage presentation as well as learning over various aspects of mathematics education: its researches, experiences, materials, information, etc. with special emphasis on achievements and trends that arose in mathematics education during the years 1996-2000 and that would contribute much to mathematics education in the new century. A number of components of the program were designed hopefully to link the scientific spirit and human wisdom for harmony on the cultural backgrounds from the East and the West, and from the North and the South,

Furthermore, all participants were invited to submit proposals for short presentation in the form of a poster, videotape or sample software. Qualified short presentations, 368 in number, were accepted and displayed. A booklet of their abstracts was distributed to all participants and visits to displays to hear the authors' comments were arranged. Non-commercial and commercial exhibitions attracted participants by including a selection of curriculum development projects, and materials from academic institutions, and also advanced products from companies.

The participants literally relaxed in Happy Hours, which included Bon-dancing, and enjoyed the Congress Tour Day, some courses of which included Japanese school visits and some others included sightseeing in famous places as Nikko, Mt. Fuji etc.

The activities of ICME 9 took place in Nippon Convention Center (NCC) , which is an international convention complex located in the core of Makuhari, and in Chiba Institute of Technology (CIT) neighboring Makuhari.

The book part of these Proceedings of ICME 9 includes, in its main body ,the address by the ICMI President and remarks by the ICMI Secretary, a record of International Round Table, the 4 Plenary Lectures, abstracts of the 52 Regular Lectures, and a summary report on each of Working Groups for Action and of Topic Study Groups. Brief reports on the ICMI Studies, the ICMI Affiliated Study Groups and the Chinese Forum are also included. On the other hand, the CD part of these Proceedings contains, in addition to files for the whole content of the book part, vivid scenes of the ceremonies and the IRT, full texts of regular lectures if available, and some scientific animation etc.

The editors wish that these Proceedings reflect the activities and atmosphere of the Congress, and can give an overview of the whole Congress, even to those who participated actively in some sections of the program, while many important sessions of their interest took place in parallel.

Lastly but not least, the editors want to thank all the people who provided reports for these Proceedings and would like to thank once more to all participants of ICME 9 who contributed to the success on the Congress, and hope that in spite of a delay of publication these Proceedings should be still useful and inspiring to the participants of ICME 9 and other people who are concerned with Mathematics Education in the 21st century.

Hiroshi Fujita

May 2003

On behalf of Editors:

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Presidential Address at the Opening Ceremony of ICME

Hyman Bass
University of Michigan, USA
President of ICMI

President Yoshikawa,
President Sugiyama,
Chairman Fujita,
Secretary Sawada,
Guests and Participants of the Ninth International Congress on Mathematical Education.

As the incoming President of the International Commission on Mathematical Instruction, it is my honor and privilege to welcome you to this important Congress, and to express the appreciation of all participants for the generous efforts of our Japanese hosts. Particular thanks are due to the International Program Committee, chaired by Professor Fujita, and the National Organizing Committee, chaired by Professor Sugiyama together with its Secretary, Professor Sawada, for their major organizational work, and to the Science Council of Japan, led by Rector Yoshikawa, for its generous support of the Congress.

This Congress represents two important and underdeveloped kinds of linkage. They are symbolized in the very name of the Congress. One is the strong connection between mathematics education on the one hand, and mathematics as a discipline on the other. The other is the international character of the Congress, which creates opportunities for building cross-cultural linkages among people concerned with educational practice and scholarship.

Mathematics as a discipline has a long history, emerging from many cultures. It is truly international, even universal, in character. Mathematicians throughout the world have a fundamentally common understanding of the nature of mathematics and of its central problems and methods. Research mathematicians working on a mathematical problem, be they in Africa, Asia, Australia, Europe, or the Americas, are part of a cohesive intellectual community that communicates fluently. Though mathematics is an enabling discipline for the other sciences, it has a theoretical core whose interest does not reside solely in these applications.

Mathematics education, in contrast, has a variable and culturally based character. This is certainly true of educational organization and practice. Education is conditioned by social goals and educational organization that vary widely in different countries. Educational research is both an applied social science and a

multidisciplinary domain of theoretical scholarship. And it too is culturally influenced. So the international assemblage of educational practitioners and researchers at this congress is much more extraordinary and significant than the corresponding gatherings of mathematicians at the International Congresses of Mathematics.

Among organizations devoted to mathematics education, the International Commission on Mathematical Instruction (ICMI), the sponsoring organization of this Congress, is distinctive because of its close organizational and structural ties to the mathematics research community. One expression of this is that ICMI has historically been presided over by research mathematicians, including such figures as Felix Klein, Hans Freudenthal, and Marshall Stone. As strong and fruitful as this tie has been, it does not imply an automatically healthy and symbiotic relationship between the mathematics and mathematics education professional communities. Indeed, the great challenges now facing mathematics education around the world demand a much deeper and more sensitive involvement of disciplinary mathematicians than we now have, both in the work of educational improvement and in research on the nature of teaching and learning. There are many things that have impeded such boundary crossing and collaboration, such as the need to reconcile language, epistemology, norms of evidence, and, in general, all of the intellectual and attitudinal challenges that face multidisciplinary research and development. ICME 9 brings together people who know and understand different things, to learn from each other, and hopefully to foster collaboration.

Let me conclude with some observations about the central place of mathematics in the school curriculum. Along with language, mathematics has always been at the core of education in all civilized societies. Today the study of mathematics occupies many students for the entire twelve years or more of their public schooling. This preeminent place of mathematical study is being seriously questioned in some quarters. It places hard and exacting demands on students. Other subjects, in the social as well as natural sciences, are making claims for a larger share of the curricular territory. Much of the skill traditionally conveyed by the study of mathematics is now considered by some people to be made obsolete because of the presence of modern computational technology.

Most people I know in mathematics and in mathematics education, myself included, feel deeply that this is a misguided and retrograde tendency. Yet we are challenged to articulate a compelling case for maintaining, and even strengthening, the study of mathematics in the schools. The traditional rationale has been a mixture of pragmatic, economic, social, intellectual, and cultural reasons. Pragmatic to learn the basic skills of arithmetic and of measurement, and the rudimentary geometric concepts and figures. Economic because of the quantitative literacy demanded by the rapidly evolving technological workplace. Social to provide the resources for responsible citizenship in a modern democracy. Intellectual since mathematics is

the enabling discipline for all of science, and it offers fundamental tools of analysis, quantitative expression, and disciplined reasoning. Cultural because mathematics exposes students to some of the most subtle and sublime achievements of the human spirit.

Yet people can still question whether these arguments suffice to justify so many years of mathematical study. Who today will be doing significant arithmetic without a calculator? How many people will ever have occasion as adults to solve a quadratic equation? Even engineers have no need to prove that their mathematical methods work. Why are our cultural arguments for the study of mathematics so different from those advocating the study of ancient languages and history?

I believe that these questions have compelling answers. But the answers must be framed in new ways, and grounded in new conceptions of what a contemporary mathematics curriculum and learning goals must look like. Articulating that new rationale for the study of mathematics in the schools, and a correspondingly new conceptualization of what such study should look like, is an important challenge to all of us. Perhaps the participants at this congress can give some thought to this question.

With that, it is now my privilege, on behalf of the International Commission on Mathematical Instruction, to join with Secretary Sawada in declaring open this Ninth International Congress on Mathematical Education.

TIME-TABLE OF ICME-9

As of June 23, 2000

	July 30 (Sun)	July 31 (Mon)	August 1 (Tue)	August 2 (Wed)	August 3 (Thu)	August 4 (Fri)	August 5 (Sat)	August 6 (Sun)
8:30								
9:00		Registration	RL (1)	RL (3)		RL (5)	PL (3)	Reports on WGAs
10:00		Opening Ceremony						
11:00			RL (2)	RL (4)		Short Presentations (1)	RL (6)	PL (4)
12:00		IRT						
13:00		Break	Break	Break		Break	Break	Closing Ceremony
14:00		Break						
15:00		PL (1)	WGA (1)	WGA (2)		Short Presentations (2)	WGA (3)	
16:00						ICMI Studies (1)		
17:00		PL (2)	TSG (1)	TSG (2)		ICMI A.S.G.	WGA (EX)	
18:00		Attraction				ICMI Studies (2)		
19:00			ICMI A.S.G.	ICMI A.S.G.		ICMI GA		
20:00		Welcome Cocktail	Happy Hour	Happy Hour		Happy Hour	Happy Hour	