

# GRAVITATIONAL $N$ -BODY PROBLEM

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VOLUME 31

# GRAVITATIONAL *N*-BODY PROBLEM

PROCEEDINGS OF IAU COLLOQUIUM No. 10  
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## INTRODUCTION

This volume contains the proceedings of the third IAU conference on the Gravitational  $N$ -Body Problem. The first IAU conference [1], six years ago, was motivated by the renaissance in Celestial Mechanics following the launching of artificial earth satellites, and was an attempt to bring to bear on the problems of Stellar Dynamics the sophisticated analytical techniques of Celestial Mechanics. That meeting was an outgrowth of the 'Summer Institutes in Celestial Mechanics' initiated by Dirk Brouwer. By the second IAU conference [2], our interest had been captured by the attempts to simulate stellar systems on the computer. Computer simulation is now an essential part of stellar dynamics; journals of computational physics have started in the United Kingdom and in the United States and symposia on computer simulation of many-body problems have become a perennial event [3, 4, 5]. Although our early hopes that the computer would 'solve' our problem have been tempered by experience, some techniques of computer simulation have now matured through five years of testing and use. A working description of the six most popular methods is appended to this volume.

During the past three years, stellar dynamicists have followed closely the developments in the related field of Plasma Physics. The contexts of Plasma and Stellar Physics are deceptively similar; at first, results from Plasma Physics were bodily transferred to stellar systems by 'changing the sign of the coupling'. We are more sophisticated and more skeptical now. Still the size and vitality of the Plasma effort commands our (sometimes envious) attention. We are grateful to John Dawson for organizing the tutorial session on methods and results from Plasma Physics which are presented in Chapter III.

From the viewpoint of stellar dynamics, stellar systems fall naturally into two classes, depending on whether or not encounters (the analog to collisions in gas kinetic theory) contribute to their dynamical evolution. Stellar Associations, Galactic Clusters, Globular Clusters and Clusters of Galaxies belong to the first class and are referred to here as 'Collisional Systems'. These were the first systems to be computer simulated; the 'classic' work was reported by Sebastian Von Hoerner in 1960 [6]. This is still the most active field, and Chapter I (which contains almost half of the contributions to these proceedings) is devoted to this topic.

A more recent development is the simulation of Galaxies, whose dynamics are dominated by collective ('Collisionless') effects. This work excites us not only because Galaxies are the most dramatic objects in the astronomer's sky, but also because the theory of collective interactions has made enormous progress in the past few years, and has produced a successful gravitational theory of the Spiral Arms. Recent developments on the Spiral Arm problem are contained in the Proceedings of IAU Colloquium

No. 38 [7], and in Chapter II where we deal with Collisionless Systems, we have not repeated that material.

The present colloquium was organized by George Contopoulos (who has been the guiding spirit of all three of the IAU Colloquia), Sverre Aarseth and myself, with help from Michel Hénon and Victor Szebehely. Dr Aarseth was also the host and local organizer at the Institute of Theoretical Astronomy, Cambridge University. I would like to thank Jane C. Ackland and Eliza Collins (and her staff of the Editorial and Publications Division of the Smithsonian Astrophysical Observatory) for editorial and secretarial assistance. Finally, at times when I questioned my ability to make sense out of one more abstruse contribution (my own not excepted), my courage was bolstered by remembering the patient but dogged tutelage I underwent at the hands of Professor Rupert Wildt at Yale University.

*December, 1970*  
*Cambridge, Mass.*

MYRON LECAR

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Computed star cluster with 500 initial members after 15 crossing times.  
*(By courtesy of S. J. Aarseth, Institute of Theoretical Astronomy, Cambridge.)*



Real star cluster M 67 photographed in blue with a 17" Schmidt telescope.  
*(By courtesy of A. N. Argue, The Observatories, Cambridge.)*