APPROXIMATE SOLUTION OF OPERATOR EQUATIONS

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Preface

One of the most important chapters in modern functional analysis is the theory of approximate methods for solution of various mathematical problems. Besides providing considerably simplified approaches to numerical methods, the ideas of functional analysis have also given rise to essentially new computation schemes in problems of linear algebra, differential and integral equations, nonlinear analysis, and so on.

The general theory of approximate methods includes many known fundamental results. We refer to the classical work of Kantorovich; the investigations of projection methods by Bogolyubov, Krylov, Keldysh and Petrov, much furthered by Mikhlin and Pol'skii; Tikhonov's methods for approximate solution of ill-posed problems; the general theory of difference schemes; and so on.

During the past decade, the Voronezh seminar on functional analysis has systematically discussed various questions related to numerical methods; several advanced courses have been held at Voronezh University on the application of functional analysis to numerical mathematics. Some of this research is summarized in the present monograph. The authors' aim has not been to give an exhaustive account, even of the principal known results.

The book consists of five chapters.

In the first chapter we study iterative processes: conditions for convergence, estimates of convergence rate, effect of round-off errors, etc. Much attention is paid to the convergence of iterative processes under conditions incompatible with the contracting mapping principle (the theory of concave operators, the role of uniformly convex norms, and so on). The second chapter studies linear problems: methods for approximate solution of linear equations, estimates for the spectral radius of a linear operator, approximate determination of eigenvalues, etc. The theory of semiordered spaces plays an important role. The third chapter considers equations with smooth nonlinear operators, employing ideas close to those of Kantorovich. Considerable attention is paid to the situations arising in approximate methods which utilize various simplified formulas. Topological methods are proposed for *a posteriori* error estimates. Much of Chapters 1 to 3 borrows from the above-mentioned advanced courses, which were given alternately by Krasnosel'skii and Rutitskii; a few sections in the first and second chapters were written by Krasnosel'skii and Stetsenko.

Chapter 4 is devoted to a systematic theory of projection methods (method of least squares, the methods of Galerkin, Galerkin–Petrov, et al.) as applied to the approximate solution of linear and nonlinear equations, and approximate determination of eigenvalues. Most of this chapter was written by G. M. Vainikko.

The fifth and last chapter considers approximate methods in a difficult field of nonlinear analysis—the theory of branching of small solutions. The authors have seen fit to present a short account of the basic theory of formal power series. This chapter was written by Zabreiko and Krasnosel'skii.

The book includes a large number of exercises, ranging from the simple to the very difficult.

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The authors