A Modern Course on

Statistical Distributions in Scientific Work

Volume 3 – Characterizations and Applications

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Volume 3 – Characterizations and Applications

A Modern Course on

Statistical Distributions in Scientific Work

Volume 3 – Characterizations and Applications

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STATISTICAL DISTRIBUTIONS IN SCIENTIFIC WORK

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of Statistical Distributions With Applications

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PREFACE

These three volumes constitute the edited Proceedings of the NATO Advanced Study Institute on Statistical Distributions in Scientific Work held at the University of Calgary from July 29 to August 10, 1974. The general title of the volumes is "Statistical Distributions in Scientific Work". The individual volumes are: Volume 1 - Models and Structures; Volume 2 - Model Building and Model Selection; and Volume 3 - Characterizations and Applications. These correspond to the three advanced seminars of the Institute devoted to the respective subject areas.

The planned activities of the Institute consisted of main lectures and expositions, seminar lectures and study group discussions, tutorials and individual study. The activities included meetings of editorial committees to discuss editorial matters for these proceedings which consist of contributions that have gone through the usual refereeing process. A special session was organized to consider the potential of introducing a course on statistical distributions in scientific modeling in the curriculum of statistics and quantitative studies. This session is reported in Volume 2. The overall perspective for the Institute is provided by the Institute Director, Professor G. P. Patil, in his inaugural address which appears in Volume 1. The Linnik Memorial Inaugural Lecture given by Professor C. R. Rao for the Characterizations Seminar is included in Volume 3.

As discussed in the Institute inaugural address, not much systematic attention has been paid to the study of statistical distributions with the modern emphasis of families, models, structures and characterizations using relevant inspiration from both statistical methodology and scientific method. The purpose of the Advanced Study Institute program was to provide an open forum with focus on recent, current and forward-looking communications on different aspects of statistical distributions arising in scientific or statistical work. The program was geared for statisticians, scientists and engineers interested in the varied work relating to statistical distributions so that they might come and learn from each other in a stimulating environment. The purpose of the Characterizations Conference was to bring together research workers investigating characterizations problems that

have motivation in scientific concepts and formulations or that have application or potential use for statistical theory.

For purposes of convenience, the Proceedings are being published in the three volumes as stated above. All together, they consist of 23 topical sections of 99 contributions of 1219 pages of research, review and exposition, in addition to a common preface in each followed by individual volume introductions. Subject indexes are also prepared at the end. Every effort has been made to keep the coverage of the volumes close to their individual titles. However, a quick glance of the three volumes will reveal that the volumewise overlaps of the topics as well as contents are not quite void! For example, several contributions appearing in Volume 2 do properly belong in Volume 3 as well. May this three volume set in its own modest way provide an example of synergism.

In order to appreciate the development and maturity of the field of statistical distributions during the last decade, these volumes may be compared with the Proceedings of the International Symposium on Classical and Contagious Discrete Distributions held at McGill in 1963--a milestone in the recognition and development of the theory and application of statistical distributions. The period between the McGill and Calgary meetings is marked by the appearance of Patil and Joshi's Dictionary and Bibliography of Discrete Distributions, Ord's and Mardia's monographs on Families of Distributions, Random Counts in Scientific Work edited by Patil in a three-volume set, and the encyclopedic four volumes on Distributions in Statistics by Johnson and Kotz.

Several participants from the McGill Symposium continued to work in the distributions area and were able to participate in the Calgary program. Moreover, topics such as the Gurland-Tripathi contribution in Volume 1 and the contributions in Volume 3 on the damage model introduced by C. R. Rao at McGill are a direct outgrowth of the pioneering results initiated at that Symposium. A number of new faces and topics were also visible at the Calgary Institute.

The realization of any institute or conference often fails to match the initial expectations and objectives of the organizers. Factors that are both logistic and psychological in nature tend to contribute to this discrepancy. Logistic difficulties include optimality problems for time and location. Other difficulties which must be attended to involve conflicting attitudes towards the importance of contributions to the proceedings.

We tried to cope with these problems by seeking active advice from a number of special advisors. The advice we received was immensely helpful in guiding our selection of the best experts in

the field to achieve as representative and balanced a picture as possible. Simultaneously, the editors together with the referees and editorial collaborators took a rather critical and constructive attitude from initial to final stages of preparation of papers by offering specific suggestions concerning the structure, content and size. These efforts of coordination and revision were intensified through editorial sessions at the Institute itself as a necessary step for the benefit of both the general readership and the participants. It is our pleasure to record with appreciation the spontaneous cooperation of the participants. Everyone went by scientific interests often at the expense of personal preferences. The Institute atmosphere became truly creative and friendly, and this remarkable development contributed to the maximal cohesion of these volumes within the limited time period available.

Clearly the volumes attempt to cover a wide spectrum of topics on the mainstream of contemporary statistical distributions. Hopefully, an alert reader will find abundant information about the present state of art and will also be able to detect prevailing trends. These studies are important in view of the ever-increasing modeling-consciousness and sophistication of the real-world studies. The availability of the computer devices is also a significant factor. One can't help wondering what course statistical distributions (and statistics in general) would have taken, had the early giants of statistical methodology had access to the modern computer! Very likely the development of statistical distributions would have speeded up and this would have provided a direction towards better collating and interweaving of statistical theory and practice. A purpose of the Institute will be served if these proceedings help provide that direction.

In retrospect, our goals were perhaps ambitious! We had close to 100 lectures and discussions during 12 days in the middle of the summer season. For several reasons, we decided that an overworked Advanced Study Institute was to be preferred to a leisurely one. First of all, gatherings of such dimension are possible only every 5-10 years. Secondly, the previous meeting of this nature occurred some 11 years back, and the subject area of statistical distributions has witnessed an unprecedented growth in this time. Thirdly, but most importantly, was the overwhelming response from the potential participants, many of whom were to come across the continents!

Some of the above remarks, which might initially appear as irrelevant as Monday morning quarterbacking, should help the reader to appreciate what one can find in these volumes and to understand (if not to wholly forgive!) any inevitable rough edges, minor duplications and inconsistencies. We very much hope that these three volumes together will provide comprehensive and

convenient reading and reference material to interested researchers, instructors and students. Each volume carries an introduction for its contents and it is hoped that these introductions will have also served a useful purpose.

In any collaborative exercise of this magnitude and nature, the unstinting support of a large number of individuals and institutions is a prerequisite for success. We are particularly grateful to the Scientific Affairs Division of NATO whose grant made the whole project feasible. Also, to the National Research Council of Canada and to the United States Army Research Office for their support, and to the various other governments and institutions whose assistance to individual members of the Institute made it a truly international gathering representing all the continents! The International Statistical Institute and the Indian Statistical Institute cosponsored the program. Our thanks go also to The Pennsylvania State University and to the University of Calgary for providing facilities. We must thank our advisors at this time too.

The success of the Institute was due, in no small measure, to the endeavors of the Local Arrangements Committee: E. G. Enns (Chairman), J. K. Wani, and P. C. Consul. We thank them for their hospitality and support. We also wish to express our sincere appreciation to Mr. M. V. Ratnaparkhi for his varied assistance during the entire project.

Miss Karen McDermid was an ever cheerful and industrious Institute secretary in the face of every adversity. Our thanks also go to the secretaries who prepared the final versions of the manuscripts. Mrs. Bonnie Henninger very ably looked after Volumes 1 and 3 in addition to the continuing chores of the Institute Director's office. Mrs. Anne Kempson took care of Volume 2 in England.

All these three volumes have been included in the ongoing NATO Advanced Study Institutes Series. They are published by the D. Reidel Publishing Company, a member of the Board of Publishers of the NATO ASI Series. It is only proper that we conclude here with our sincere thanks to both the publisher and the NATO Scientific Affairs Division for these cooperative arrangements.

January 31, 1975

G. P. Patil
S. Kotz
J. K. Ord

INTRODUCTION TO VOLUME 3

It may be a safe prediction that future historians of statistics will debate for many years the question of the place, impact and even appropriateness of characterization theorems in the framework of mathematical statistics and its development. The opinions heard today range from "an interesting mathematical subdiscipline which has little use in statistical methodology" to "an indispensable tool for understanding the missing links between the mathematical structure of statistical distributions and the actual behavior of real world random phenomena". An analogy with the position of Shannon's information theory within the general engineering discipline of communication theory comes to mind. It is reasonable to assert that although both these theories do not offer a well defined method for solving real world problems, nevertheless they guide the engineer or the statistician as the case may be toward a proper scientific solution.

A highlight in the development and consolidation of the characterization field is no doubt the partnership between Kagan, Linnik and Rao which has resulted in the appearance of their volume on "Characterization Theorems in Mathematical Statistics" almost simultaneously in Russian, English and French. The monographs of Lukacs and Laha have also played a prominent role.

Characterization theorems are located on the borderlines of scientific modeling, probability theory and mathematical statistics. They utilize numerous classical tools of mathematical analysis such as the advanced theory of complex variables, differential equations of various types, theory of series and last but not least, the theory of functional equations. They therefore attracted the attention of numerous mathematicians eager to use their mathematical tools to solve applied problems without necessarily being attracted to the field of applications. This state of affairs has significantly advanced the field of characterization theorems; however, the advancement was not always directed towards the understanding of the "mysteries" in the structure of various distributions and sometimes the beauty of the mathematical result was sufficient to justify its publication even if the statistical contribution was nominal.

It is on this background that the organizers of the Calgary meeting decided to have the Characterizations Seminar immediately after the Seminar devoted to the applications of statistical distributions and before the Seminar on the contents of a Modern Course on Statistical Distributions. The idea was to achieve the maximal possible mixing and interaction between the mathematically oriented contributors to the field of characterization theorems and the most outspoken proponents of some graphical and ad hoc applied methods in statistical investigations. This confrontation will hopefully result in a reevaluation of the participants' philosophy and a reexamination of priorities. It may be mentioned here that we have already seen a first successful indication of the collaboration between the representatives of different parts of the spectrum which resulted in a paper by Aczél developed during the Conference in close collaboration with Talwalker. We hope to witness many more joint efforts in the coming years.

The purpose of the present volume has been to present a picture of the state of the art in the field of characterizations in 1974. It consists of 9 sections of 37 contributions made by several prominent and upcoming researchers active in the field. The interested reader will supplement this material by characterizations-relevant contributions appearing in Volumes 1 and 2, such as those by Rukhin, Patil-Seshadri, Patil-Boswell-Friday, Csörgö-Seshadri-Yalovsky, Katti-McDonald, Chatfield, Mosimann, Talwalker and a few others. Initially prepared for the Institute purposes, the comprehensive survey paper by Kotz in the 1974 International Statistical Review will also provide a good supplementary reading. All these papers together should provide a broad coverage of both theory and applications of present day characterizations of distributions.

May these volumes help stimulate and consolidate the present and future growth of the subject in directions that are fruitful.

January 31, 1975

G. P. Patil
S. Kotz
J. K. Ord