

ENCYCLOPAEDIA OF MATHEMATICS

Volume 2

ENCYCLOPAEDIA OF MATHEMATICS

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An updated and annotated translation of the Soviet
'Mathematical Encyclopaedia'



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PREFACE

This ENCYCLOPAEDIA OF MATHEMATICS aims to be a reference work for all parts of mathematics. It is a translation with updates and editorial comments of the Soviet *Mathematical Encyclopaedia* published by ‘Soviet Encyclopaedia Publishing House’ in five volumes in 1977 – 1985. The annotated translation consists of ten volumes including a special index volume.

There are three kinds of articles in this ENCYCLOPAEDIA. First of all there are survey-type articles dealing with the various main directions in mathematics (where a rather fine subdivision has been used). The main requirement for these articles has been that they should give a reasonably complete up-to-date account of the current state of affairs in these areas and that they should be maximally accessible. On the whole, these articles should be understandable to mathematics students in their first specialization years, to graduates from other mathematical areas and, depending on the specific subject, to specialists in other domains of science, engineers and teachers of mathematics. These articles treat their material at a fairly general level and aim to give an idea of the kind of problems, techniques and concepts involved in the area in question. They also contain background and motivation rather than precise statements of precise theorems with detailed definitions and technical details on how to carry out proofs and constructions.

The second kind of article, of medium length, contains more detailed concrete problems, results and techniques. These are aimed at a smaller group of readers and require more background expertise. Often these articles contain more precise and refined accounts of topics and results touched upon in a general way in the first kind of article.

Finally, there is a third kind of article: short (reference) definitions.

Practically all articles (all except a few of the third kind) contain a list of references by means of which more details and more material on the topic can be found. Most articles were specially written for the encyclopaedia and in such cases the names of the original Soviet authors are mentioned. Some articles have another origin such as the *Great Soviet Encyclopaedia (Bol'shaya Sovetskaya Entsiklopediya* or BSE).

Communication between mathematicians in various parts of the world has certainly greatly improved in the last decennia. However, this does not mean that there are so-to-speak ‘one-to-one onto’ translations of the terminology, concepts and tools used by one mathematical school to those of another. There also are varying traditions of which questions are important and which not, and what is considered a central problem in one tradition may well be besides the point from the point of view of another. Even for well-established areas of mathematical inquiry, terminology varies across languages and even within a given language domain. Further, a concept, theorem, algorithm, . . . , which is associated with one proper name within one tradition may well have another one in another, especially if the result or idea in question was indeed discovered independently and more-or-less simultaneously. Finally, mathematics is a very dynamic science and much has happened since the original articles were finalized (mostly around 1977). This made updates desirable (when needed). All this, as well as providing additional references to Western literature when needed, meant an enormous amount of work for the

board of experts as a whole; some indeed have done a truly impressive amount of work. I must stress though that I am totally responsible for what is finally included and what is not of all the material provided by the members of the board of experts.

Many articles are thus provided with an editorial comment section in a different and somewhat smaller typeface. In particular, these annotations contain additional material, amplifications, alternative names, additional references, Modifications, updates and other extra material provided by the original Soviet authors (not a rare occurrence) have been incorporated in the articles themselves.

The final (10-th) volume of the *ENCYCLOPAEDIA OF MATHEMATICS* will be an index volume. This index will contain all the titles of the articles (some 6600) and in addition the names of all the definitions, named theorems, algorithms, lemmas, scholia, constructions, . . . , which occur in the various articles. This includes, but is by no means limited to, all items which are printed in bold or italic. Bold words or phrases, by the way, always refer to another article with (precisely) that title.

All articles have been provided with one or more AMS classification numbers according to the 1980 classification scheme (not, for various reasons, the 1985 revision), as have all items occurring in the index. A phrase or word from an article which is included in the index always inherits all the classification numbers of the article in question. In addition, it may have been provided with its own classification numbers. In the index volume these numbers will be listed with the phrase in question. Thus e.g. the Quillen – Suslin theorem of algebraic K -theory will have its own main classification numbers (these are printed in bold; in this case that number is 18F25) as well as a number of others, often from totally different fields, pointing e.g. to parts of mathematics where the theorem is applied, or where there occurs a problem related to it (in this case e.g. 93D15). The index volume will also contain the inversion of this list which will, for each number, provide a list of words and phrases which may serve as an initial description of the ‘content’ of that classification number (as far as this *ENCYCLOPAEDIA* is concerned). For more details on the index volume, its structure and organisation, and what kind of things can be done with it, cf. the (future) special preface to that volume.

Classifying articles is a subjective matter. Opinions vary greatly as to what belongs where and thus this attempt will certainly reflect the tastes and opinions of those who did the classification work. One feature of the present classification attempt is that the general basic concepts and definitions of an area like e.g. 55N (Homology and Cohomology theories) or 60J (Markov processes) have been assigned classification numbers like 55NXX and 60JXX if there was no finer classification number different from ...99 to which it clearly completely belongs.

Different parts of mathematics tend to have differences in notation. As a rule, in this *ENCYCLOPAEDIA* in a given article a notation is used which is traditional in the corresponding field. Thus for example the (repeated index) summation convention is used in articles about topics in fields where that is traditional (such as in certain parts of differential geometry (tensor geometry)) and it is not used in other articles (e.g. on summation of series). This pertains especially to the more technical articles.

For proper names in Cyrillic the British Standards Institute transcription system has been used (cf. *Mathematical Reviews*). This makes well known names like S.N. Bernstein come out as Bernshteĭn.

In such cases, especially in names of theorems and article titles, the traditional spelling has been retained and the standard transcription version is given between brackets.

Ideally an encyclopaedia should be complete up to a certain more-or-less well defined level of detail. In the present case I would like to aim at the completeness level whereby every theorem,

concept, definition, lemma, construction which has a more-or-less constant and accepted name by which it is referred to by a recognizable group of mathematicians occurs somewhere, and can be found via the index. It is unlikely that this completeness ideal will be reached with this present ENCYCLOPAEDIA OF MATHEMATICS, but it certainly takes substantial steps in this direction. Everyone who uses this ENCYCLOPAEDIA and finds items which are not covered, which, he feels, should have been included, is invited to inform me about it. When enough material has come in this way supplementary volumes will be put together.

The ENCYCLOPAEDIA is alphabetical. Many titles consist of several words. Thus the problem arises how to order them. There are several systematic ways of doing this of course, for instance using the first noun. All are unsatisfactory in one way or another. Here an attempt has been made to order things according to words or natural groups of words as they are daily used in practice. Some sample titles may serve to illustrate this: **Statistical mechanics, mathematical problems in; Lie algebra; Free algebra; Associative algebra; Absolute continuity; Abstract algebraic geometry; Boolean functions, normal forms of.** Here again taste plays a role (and usages vary). The index will contain all permutations. Meanwhile it will be advisable for the reader to try out an occasional transposition himself. Titles like K -theory are to be found under K , more precisely its lexicographic place is identical with 'K theory', i.e. '-' = 'space' and comes before all other symbols. Greek letters come before the corresponding Latin ones, using the standard transcriptions. Thus χ^2 -distribution (chi-squared distribution) is at the beginning of the letter C. A \star as in C^* -algebra and \star -regular ring is ignored lexicographically. Some titles involve Greek letters spelled out in Latin. These are of course ordered just like any other 'ordinary' title.

This volume has been computer typeset using the (Unix-based) system of the CWI, Amsterdam. The technical (mark-up-language) keyboarding was done by Rosemary Daniëls, Chahrzade van 't Hoff and Joke Pesch. To meet the data-base and typesetting requirements of this ENCYCLOPAEDIA substantial amounts of additional programming had to be done. This was done by Johan Wolleswinkel. Checking the translations against the original texts, and a lot of desk editing and daily coordination was in the hands of Rob Hoksbergen. All these persons, the members of the board of experts, and numerous others who provided information, remarks and material for the editorial comments, I thank most cordially for their past and continuing efforts.

The original Soviet version had a print run of 150,000 and is completely sold out. I hope that this annotated and updated translation will turn out to be comparably useful.