The General Theory of Alternating Current Machines:

Application to Practical Problems

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Preface

The book on *The General Theory of Electrical Machines*, by B. Adkins, which was published in 1957, has been well received, as a manual containing the theories on which practical methods of calculating machine performance can be based, and as a text-book for advanced students. Since 1957, many important developments have taken place in the practical application of electrical machine theory. The most important single factor in the development has been the increasing availability of the digital computer, which was only beginning to be used in the solution of machine and power system problems in 1957. Since most of the recent development, particularly that with which the authors have been concerned, has related to a.c. machines, the present book, which is in other respects an up-to-date version of the earlier book, deals primarily with a.c. machines.

The second chapter on the primitive machine does deal to some extent with the d.c. machine, because the cross-field d.c. generator serves as an introduction to the two-axis theory and can be used to provide a simple explanation of some of the mathematical methods. The equations also apply directly to a.c. commutator machines.

The use of the word 'general' in the title has been criticized. It was never intended to imply that the treatment was *comprehensive* in the sense that every possible type of machine and problem was dealt with. The word is used in the sense that the theory *can* be applied to all types of machine and all conditions of operation. Strictly of course the theory does not apply exactly to any machine, but only to an idealized model, which is similar to the practical machine. The whole range of machines can be divided into three categories,

1. Those for which the theory can give an accurate prediction of its behaviour from design details.
2. Those where there are larger discrepancies but where the theory helps in obtaining an understanding of the problem.
3. Those for which the theory cannot be usefully employed.

The great majority of synchronous and induction machines in use at the present time fall into the first category, mainly because the design process aims at the elimination of harmonics and other factors which constitute the difference between the idealized and the practical machine.

Although the book is still mainly concerned with the two-axis theory and its development, the generalization has been extended to cover other reference frames, particularly that in which the variables are the actual phase currents in an a.c. armature. In recent years an increasing number of special machines, like linear motors, inductor alternators, reluctance motors or new types of change-pole motors, are being used. Sometimes the two-axis theory cannot be used or would be too inaccurate, but a theory based on the phase equations would be satisfactory. Without the computer a solution would usually not be practicable, but the equations can now be readily programmed for computation. Saturation and eddy current effects can often be handled within the framework of the theory, as explained in Chapter 10, but in some of the special machines the results would be too inaccurate. The authors nevertheless make no apology for continuing to use the word *general*, with the above qualifications.

In the interest of greater mathematical rigour, Laplace transforms are introduced when solutions of linear equations are derived. However, the Heaviside formulation, in which the symbol p is regarded as an operator, is used both in the initial statements of equations, linear and non-linear, and in the application of operational transfer functions and equivalent circuits.

Another change, which is a result of increasing use of computers, is that a short explanation is given in Chapter 5 of some of the modern control theories for which the equations are expressed in state variable form. The methods are particularly applicable to systems with multiple controls and to those for which the equations are non-linear. Some examples of their application to step-by-step computations are given in Chapter 9.

Some changes of notation have been made to conform to the new international specifications. The term *phasor* is used instead of *vector* for the complex number representing a sinusoidal quantity, while the word *vector* is used for a set of state-space variables.

Over a period of twenty years, the micro-machine equipment at Imperial College has been used by a team of research workers to study problems relating to a.c. machines and the external systems and devices associated with them. They have also had the opportunity of studying the results of tests made on large machines in factories or power stations. The programme of work at Imperial College is only one of many such activities round the world, but it has been found convenient to use the results of this programme, which are recorded in a number of I.E.E. papers, for many of the practical applications of the methods described in the book. As stated above, the theory is of general application, but it is only necessary to give a few selected illustrations. A bibliography giving references to all the work on the subject would be unduly long, but many other publications can be found from bibliographies of the listed papers.

For the theoretical derivations, the plan is adopted that a full explanation is given for the basic theory and the most important formulas. On the other hand, for the theory relating to many of the practical applications, for which the derivations are explained in the quoted references, only abbreviated explanations are given.

The book is intended for students and others who already have some knowledge of electrical machines, since it does not include any description of their construction. The mathematics required includes complex algebra, matrix algebra, Laplace transforms and a few other matters which are explained briefly in the text.

Contents

Introduction

The purpose of the book is to present a general theory of rotating electro-magnetic machines, applicable to all the normal types of machine and to all conditions of operation, and is consequently more fundamental and of wider application than the usual theories given in the standard textbooks. The theory applies to all machines in which alternate magnetic poles are formed round a cylindrical surface.

An analytical study of electrical machines consists of two parts:

1. Determination of the basic characteristics expressed by a number of quantities known as the machine *parameters*. In making calculations the values are often assumed to be constant, and the parameters are then referred to as *constants*.
2. Calculation from the constants of the performance of the machine under given external conditions.

The term 'theory of electrical machines', as interpreted in this book, refers only to the second part. The theory starts with an idealized machine, the properties of which are expressed by known constants, and provides a means of calculating its performance. For the purposes of the theory, the constants, which are essentially resistances and inductances, must be carefully defined, and the principles underlying their calculation clearly stated. Details of the methods of calculating them, with which the first of the above two parts is concerned, although very important for practical design work, form a separate subject.

In the usual textbook theories of electrical machines each type of machine is dealt with on its own merits with little reference to other types, and simple methods of analysis are developed by means of which the performance under specified conditions can be calculated. In these theories the main emphasis is on steady operation, and they lead to graphical or analytical methods of calculation. For a.c. machines, phasor diagrams are very widely used. The standard approach has the disadvantage that a completely fresh start has to be made when it is necessary to analyse a new type of machine or to deal with unbalanced or transient conditions.

It is interesting to survey the historical development of the theory of electrical machines. The early theories of a.c. machines were based on the phasor diagram and were worked out by geometrical constructions on a drawing-board. There followed a search for equivalent circuits, which finally led to the acceptance of a few standard circuits selected from a large number of possible ones. The next important development was the introduction of complex numbers in what was known as the 'symbolic method' or the 'j-method'. At that time, however, the algebraic method was not introduced in its own right, but only as an auxiliary process to assist in working out the phasor diagrams or equivalent circuits. The modern methods involve a new approach to the subject. In the modern theory the algebraic equations are accepted as the fundamental means of expression, and phasor diagrams and equivalent circuits become merely devices leading to alternative methods of solution applicable only to special cases. The use of equations is in line with the accepted methods of circuit theory, and leads to a general theory of electrical machines which embraces all types and all conditions of operation.

The fundamental set of equations is derived for an idealized two-pole machine, which is approximately equivalent to the actual machine, in accordance with certain well-defined assumptions. In general they are differential equations in which an applied voltage is equated to the sum of several component voltages which depend on the currents, or an applied torque is equated to the sum of component torques. In d.c. machines the equations relating the actual currents with the voltages and torque are usually in a convenient form for practical solution, but for most problems in a.c. machines the equations tend to be complicated and difficult to solve, although a solution is possible with a digital computer.

However, for the great majority of practical a.c. machines, including normal induction and synchronous machines, a great simplification is obtained by expressing the equations in a new reference frame and introducing certain fictitious currents and voltages which are different from but are related to the actual ones. The fictitious currents can have a physical meaning in that they can be considered to flow in fictitious windings acting along two axes at right angles, called the direct and quadrature axes. In this way a 'two-axis theory' of a.c. machines is developed, and the equations so obtained are found to correspond very closely to those of d.c. machines.

Because of its practical importance, a large part of the book is devoted to applications of the two-axis theory, particularly to synchronous and induction machine problems. The first step in its development was Blondel's 'two-reaction theory' of the steady-state operation of the salient-pole synchronous machine. The method was examined in detail by Doherty and Nickle, who published a series of five important papers [5]. A paper by West on 'The Cross-field Theory of Alternating Current Machines' [2] assumed without proof that a rotating cage winding is equivalent to a d.c. armature winding with two short-circuited pairs of brushes. A very valuable contribution to the subject was made by Park in a set of three papers [3, 4 and 6]. These papers not only develop the general two-axis equations of the synchronous machine, but they indicate how the equations can be applied to many important practical problems. Park's transformation provides the most important fundamental concept in the development of Kron's generalized theory, which was first published in a series of papers in The General Electric Review, and later in a book [8]. Many more recent books and papers on the subject are referred to in the bibliography.

After discussing some general matters in Chapter 1, the equations of the primitive machine and some simple applications are considered in Chapter 2. Chapters 3 and 4 develop the theory of the a.c. machine in terms both of the actual armature variables and of the two-axis variables. Chapters 5 and 6 set down some fundamental matters relating to methods of analysis and automatic control and Chapters 7 to 11 are devoted to the application of the two-axis theory to many practical problems arising in the application of synchronous and induction machines. Chapter 12 discusses problems relating to other less common types of machine, including some to which the two-axis method is not applicable. Four short appendices consider some special matters which arise in applying some of the well-known mathematical methods to machine problems.

When satisfactory equations can be formulated and sufficiently accurate parameters can be determined, – two matters of equal importance – a solution can be obtained, either by a digital computation or by an analytical method. An analytical method can often lead to a general result, whereas a computation can only apply to a particular numerical case. The analytical treatments are therefore still of considerable importance. The accuracy obtained depends on the error involved in the idealization of the machine on which the formulation is based and in the assumptions made in calculating the parameters.

Terminology and notation

The terminology and notation follows as far as possible the recommendations of recent international standards. The sign conventions are chosen, as befits a general theory, so that the equations apply equally to generators and motors. On the electrical side, they agree with the established conventions of circuit theory, as explained on p. 11.

The per-unit system is used extensively and extra care has been taken with the explanations, both in the main text and in Section 13.4.

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R. G. Harley.