Probability Theory and Applications

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Contents

Preface

Probability theory and its applications represent a discipline of fundamental importance to nearly all people working in the high-technology world that surrounds us. There is increasing awareness that we should ask not "Is it so?" but rather "What is the probability that it is so?" As a result, most colleges and universities require a course in mathematical probability to be given as part of the undergraduate training of all scientists, engineers, and mathematicians.

This book is a text for a first course in the mathematical theory of probability for undergraduate students who have the prerequisite of at least two, and better three, semesters of calculus. In particular, the student must have a good working knowledge of power series expansions and integration. Moreover, it would be helpful if the student has had some previous exposure to elementary probability theory, either in an elementary statistics course or a finite mathematics course in high school or college. If these prerequisites are met, then a good part of the material in this book can be covered in a semester (15-week) course that meets three hours a week.

The purpose of a course in probability is to provide the student with an understanding of randomness and give some practice in the use of mathematical techniques to analyze and interpret the effects of randomness in experiments and natural phenomena. In addition, a major aim of a first course in probability theory is to give many examples and applications relating to a wide variety of situations.

The chapters of this book make up a list of topics that are both important and reasonably possible to offer in a first course. There is less disagreement in the topics to be covered in a probability course than, say, in a statistics course. In any case, instructors should feel free to experiment with the order and emphasis given to these topics to find the arrangement most suitable for the interests of their classes. A typical arrangement might be as follows.

Chapter 1 on the algebra of sets and the axioms of probability is covered in the first week (week no. 1). Chapter 2 on the theorems of probability, combinatorial analysis, conditional probability and independence, and Bayes' theorem is covered in the second week (week no. 2). (If the students have not had an elementary introduction to probability previously, then an extra week should be spent on Chapter 2, and as compensation one week should be removed from the end of the course.)

The next three weeks can be spent on Chapter 3, namely, week no. 3 on random variables and probability mass functions, week no. 4 on distribution functions and probability density functions, and week no. 5 on random sampling and change of variable. The change-of-variable section is the critical section in the book. Although this section is strictly nothing more than the technique of transforming variables in calculus, some of the students will master this section and others will not. The ones that do are the A students, and the rest of the semester is a coast downhill for them. The other students will still have to work hard.

One week (week no. 6) is enough for Chapter 4 if only the first two sections (applications of mathematical expectation and the Chebyshev inequality) are covered, and the third section (applications to operations research) is omitted.

Three weeks (no. 7, 8, 9) can be spent on Chapter 5, which covers multivariate distributions. Weeks no. 7 and 8 would be spent on the sections on joint, marginal, and conditional distributions, and week no. 9 on the sections on expectation, covariance and correlation, regression curves, and the law of large numbers. The last section, change of several variables, would be omitted.

One week (week no. 10) is enough for Chapter 6 if the first two sections (probability generating function, gamma functions and beta functions) are omitted, and only the last two sections (moment generating function, applications) are covered.

The ten weeks up to now have prepared the student with the mathematical techniques required to handle the material in Chapters 7 and 8. These two chapters, which cover the basic distributions of probability theory, will provide the student with the concepts needed to apply his knowledge to real-world problems. Chapters 7 and 8 represent the fruits of the probability course, and the sections are organized so as to bring out the interrelationships among the various probabilistic models.

If its last section (negative binomial distribution) is omitted, and its first section (review of combinatorial methods) is gone over rapidly, then Chapter 7 can be covered in two weeks (weeks no. 11 and 12). The sections covered would be on uniform distribution, hypergeometric distribution, the Bernoulli process and binomial distribution, the Poisson process and Poisson distribution, and geometric and Pascal distributions. The final three weeks (no. 13, 14, and 15) would be spent on Chapter 8, with the final three sections (gamma distribution; beta distribution; and chi-square, F, and t distributions) omitted. The sections covered in Chapter 8 would be on uniform distribution, the Cauchy distribution, exponential and Erlang distributions, sums of random variables, normal distribution, central limit theorem, reproduction properties of the normal distribution, and bivariate normal distribution. Time does not allow for the final chapter of the book, Chapter 9 on regression and correlation, to be included in a one-semester course.

Exercises for the student are given at the end of virtually all sections, and they should be assigned and graded at least on a weekly basis in order to monitor the progress of the students in an intensive course of this kind. Because the material is difficult and involves a lot of homework on the part of the student, it is important that the classes be small and a close student–teacher relationship be maintained. However, the payoff is large. The student will not only develop a better understanding of calculus with its attendant skills, but will also be well prepared for subsequent courses in stochastic processes, time-series analysis, and mathematical statistics, as well as for courses in specific professional areas. Additional exercises appear in Appendix A.

A word might be said about the use of computers in a probability course. In the standard fifteen-week course with three lectures per week, it seems that the student has enough to do to master the calculus techniques for studying probability as given in this book. However, if an additional two-hour per week computer laboratory session is added to the course, then the computer techniques acquired would be invaluable. A companion volume, *Computer Laboratory Manual on Probability,* is in preparation to go with this volume for such an expanded course.

The University of Tulsa is at the center of a high-technology area with major industrial enterprises and research laboratories in petroleum, chemicals, aerospace, computers, electronics, and precision machinery. The College of Engineering and Applied Sciences at the university attracts many serious students and professional engineers who expect and want difficult courses such as this one. They also expect the attendant computer laboratory experience in order to translate theory into practice. The origins of this book go back to the time when the author taught this course in the Mathematics Department of the Massachusetts Institute of Technology with Professors George P. Wadsworth and Joseph G. Bryan. I learned much in working with such distinguished mathematicians, especially regression analysis, and to them I own my sincere thanks and deep appreciation. The value of this course can be judged by the fact that many of the students who took this course years ago have achieved recognition in probability theory and statistics, and hold high positions in universities, industry, and government. More importantly, several have stated that their first introduction to probability theory led them into endeavors that gave them much personal enjoyment and a sense of fulfillment.

I want to thank Terry Saunders for her excellent and caring work in conveying the various manuscripts to such beautiful form on the word processor.