

Operators and Function Theory

edited by

S. C. Power

Department of Mathematics,
University of Lancaster, Lancaster, U.K.



D. Reidel Publishing Company

Dordrecht / Boston / Lancaster

Published in cooperation with NATO Scientific Affairs Division

Proceedings of the NATO Advanced Study Institute on
Operators and Function Theory
Lancaster, U.K.
July 16-26, 1984

Library of Congress Cataloging in Publication Data

NATO Advanced Study Institute on Operators and Function Theory (1984:
Lancaster, Lancashire)
Operators and function theory.

(NATO ASI series. Series C, Mathematical and physical sciences; v. 153)
"Proceedings of the NATO Advanced Study Institute on Operators and
Function Theory, Lancaster, U.K., July 16–26, 1984"—T.p. verso.

Sponsored by the NATO Science Committee.

1. Operator theory—Congresses. 2. Analytic functions—Congresses.
I. Power, S. C. II. NATO Science Committee. III. Title. IV. Series.
QA329.N38 1984 515.7'24 85—2390
ISBN-13: 978-94-010-8871-8 e-ISBN-13: 978-94-009-5374-1
DOI: 10.1007/978-94-009-5374-1

Published by D. Reidel Publishing Company
P.O. Box 17, 3300 AA Dordrecht, Holland

Sold and distributed in the U.S.A. and Canada
by Kluwer Academic Publishers,
190 Old Derby Street, Hingham, MA 02043, U.S.A.

In all other countries, sold and distributed
by Kluwer Academic Publishers Group,
P.O. Box 322, 3300 AH Dordrecht, Holland

D. Reidel Publishing Company is a member of the Kluwer Academic Publishers Group

All Rights Reserved

© 1985 by D. Reidel Publishing Company, Dordrecht, Holland.

Softcover reprint of the hardcover 1st edition 1985

No part of the material protected by this copyright notice may be reproduced or utilized
in any form or by any means, electronic or mechanical, including photocopying, recording
or by any information storage and retrieval system, without written permission from the
copyright owner.

TABLE OF CONTENTS

PREFACE	vii
LIST OF PARTICIPANTS	ix
PROGRAMME OF THE INSTITUTE	xiii
<u>MAIN ARTICLES</u>	
BLOCH FUNCTIONS: THE BASIC THEORY J. M. Anderson	1
A SURVEY OF SOME RESULTS ON SUBNORMAL OPERATORS J. B. Conway	19
OPTIMIZATION, ENGINEERING, AND A MORE GENERAL CORONA THEOREM J. W. Helton	39
MINIMAL FACTORIZATION, LINEAR SYSTEMS AND INTEGRAL OPERATORS M. A. Kaashoek	41
HA-PLITZ OPERATORS: A SURVEY OF SOME RECENT RESULTS N. K. Nikol'skii	87
STOCHASTIC PROCESSES, INFINITESIMAL GENERATORS AND FUNCTION THEORY B. Øksendal	139
PARACOMMUTATORS AND MINIMAL SPACES J. Peetre	163
DECOMPOSITION THEOREMS FOR BERGMAN SPACES AND THEIR APPLICATIONS R. Rochberg	225

OPERATOR-THEORETIC ASPECTS OF THE NEVANLINNA-PICK INTERPOLATION PROBLEM D. Sarason	279
CYCLIC VECTORS IN BANACH SPACES OF ANALYTIC FUNCTIONS A. L. Shields	315
INTERPOLATION BY ANALYTIC MATRIX FUNCTIONS N. J. YOUNG	351

PREFACE

In the modern study of Hilbert space operators there has been an increasingly subtle involvement with analytic function theory. This is evident in the analysis of subnormal operators, Toeplitz operators and Hankel operators, for example. On the other hand the operator theoretic viewpoint of interpolation by analytic functions is a powerful one. There has been significant activity in recent years, within these enriching interactions, and the time seemed right for an overview of the main lines of development.

The Advanced Study Institute 'Operators and Function Theory' in Lancaster, 1984, was devoted to this, and this book contains expanded versions (and one contraction) of the main lecture programme. These varied articles, by prominent researchers, include, for example, a survey of recent results on subnormal operators, recent work of Soviet mathematicians on Hankel and Toeplitz operators, expositions of the decomposition theory and interpolation theory for Bergman, Besov and Bloch spaces, with applications for special operators, the Krein space approach to interpolation problems, .. and much more. It is hoped that these proceedings will bring all this lively mathematics to a wider audience.

Sincere thanks are due to the Scientific Committee of the North Atlantic Treaty Organisation for the generous support that made the institute possible, and to the London Mathematical Society and the British Council for important additional support. Warm thanks also go to Barry Johnson and the L.M.S. for early guidance, and to my colleague Graham Jameson for much organisational support.

Lancaster, November 1984

Stephen Power

LIST OF PARTICIPANTS

- Adams, G., Math. Dept., Indiana University, Bloomington, IN 47401,
U.S.A.
- Albrecht, E., Math. Inst. Univ. des Saarlandes, D-6600, Saarbrücken,
W. Germany
- Alpay, S., Dept. of Math., Middle East Technical Univ., Ankara,
Turkey
- Amar, E., U.E.R. de Mathématiques et Informatique, Univ. de
Bordeaux I, 351 Cours de la Libération, 33405, Talence, France
- Anderson, J.M., Dept. of Math., University College, London WC1
E6BT, U.K.
- Arazy, J., Dept. of Math., Univ. of Haifa, Haifa, Israel
- Arveson, W., Dept. of Math., Univ. of California, Berkeley, Ca
94720, U.S.A.
- Atzmon, A., Dept. of Math., Technion, Haifa, Israel
- Axler, S., Dept. of Math., Michigan State Univ., E. Lansing, MI
48824, U.S.A.
- Aziz, S.A., Istituto Nazionale di Fisica Nucleare, Lab. Nazionali
di Frascati, Casella Postale 13-00044, Frascati, Roma, Italy
- Bailey, G.H., Dept. of Math., Univ. of Lancaster, Lancaster LA1
4YL, U.K.
- Ball, J.A., Dept. of Math., Virginia Tech., Blacksburg, VA 24061,
U.S.A.
- Bart, H., Econometrisch Inst., Erasmus Univ. Rotterdam, Postbus
1738, 3000 DR Rotterdam, Netherlands
- Békollé, D., Dept. of Math., Univ. de Bretagne Occidentale, 6
Avenue le Gorgeu, 29283 Brest Cedex, France
- Berg, I.D., Dept. of Math., Univ. of Illinois, Urbana, IL 61801,
U.S.A.
- Berger, C., Dept. of Math., Lehman College, City Univ of New York,
Bronx, N.Y. 10468
- Bonsall, F.F., 18 Rossett Park Road, Harrogate, HG2 9NP, U.K.
- Brannan, D.A., Faculty of Math., The Open Univ., Milton Keynes
MK7 6AA, U.K.
- Brennan, J.E., Dept. of Math., Univ. of Kentucky, Lexington, KY
40506, U.S.A.

- Brown, L., Dept. of Math., Wayne State Univ., Detroit, MI 48202,
U.S.A.
- Chauvenheid, P.G., Inst. Math. de l'Universite, Avenue des
Tilleuls 15, B-4000 Liège, Belgium.
- Chevreau, B., U.E.R. de Math. et Informatique, Univ. de Bordeaux
I, 351 Cours de la Liberation, 33405 Talence, France
- Clancey, K.F., Dept. of Math., Univ. of Georgia, Athens, Ga 30602,
U.S.A.
- Clunie, J., Faculty of Math., The Open Univ., Milton Keynes, MK7
6AA, U.K.
- Coburn, L.A., Dept. of Math., 106 Diefendorf Hall, SUNY at Buffalo,
N.Y 14214-3093
- Conway, J., Dept. of Math., Indiana Univ., Bloomington IN 47405,
U.S.A.
- Cowen, C.C., Dept. of Math., Purdue Univ., West Lafayette, IN
47907, U.S.A.
- Davidson, K.R., Dept. of Math., Univ. of Waterloo, Waterloo,
Ontario, Canada N2L 3G1
- Dixon, P.G., Dept. of Pure Math., The Univ. Sheffield S3 7RH, U.K.
- Douglas, R.G., Dept. of Math., SUNY at Stony Brook, NY 11794, U.S.A.
- Dowson, H.R., Dept. of Math., Univ. of Glasgow, Univ. Gardens,
Glasgow G12 8QW, U.K.
- Doyle, J.C., 1200 Nicollet Mall 618, Minneapolis, MN 55403, U.S.A.
- Dudziak, J., Dept. of Math., Bucknell Univ., Lewisburg, PA 17837,
U.S.A.
- Duncan, J., Dept. of Math., Univ. of Stirling, Stirling, U.K.
- Dym, H., Dept. of Theoretical Math., The Weizmann Inst. of Science,
Rehovot, Israel 76100
- Edwards, D.A., Math. Inst., 24-29 St. Giles, Oxford OX1 3LB, U.K.
- Fillmore, P., Dept. of Math., Dalhousie Univ., Halifax, N.S.,
Canada
- Fisher, S., Dept. of Math. Northwestern Univ., Evanston, IL 60201,
U.S.A.
- Francis, B., Dept. of Math., Univ. of Waterloo, Waterloo, Ontario,
Canada N2L 3G1
- Ghahramani, F., Dept. of Math., Univ. of Teacher Education, 49
Mofateh Ave., Tehran, Iran
- Giotopoulos, S., Dept. of Math., Univ. of Athens, Athens, Greece
- Gohberg, I.C., School of Math. Sciences, Tel Aviv Univ., Ramat
Aviv, Israel
- Gokturk, Z., Dept. of Math., Bogazici Univ., P.K.2. Bebek-Istanbul
Turkey
- Gomes, M., Dept. of Stat., Faculty of Sci., Univ. of Lisbon, 58
Rue da Escola Politécnica, 1294 Lisboa Codex, Portugal
- Gorkin, P., Dept. of Math., Bucknell Univ., Lewisburg, PA 17837,
U.S.A.
- Halmos, P., Dept. of Math., Indiana Univ., Bloomington, IN 47405,
U.S.A.
- Haslinger, F., Inst. fur Mathematik, Univ. Wien, Strudlhofg. 4,
A-1090, Wien, Austria

- Helton, W., Dept. of Math., Univ. of California at San Diego, La Jolla, CA 92037, U.S.A.
- Holland, C., Office of Naval Res., 800 N Quincy St., Arlington, VA 22217, U.S.A.
- Holland, F., Dept. of Math., University College, Cork, Ireland
- Hoppenwasser, A., Dept. of Math., Univ. of Alabama, AL 35486, U.S.A.
- Jameson, G.J.O., Dept. of Math., University of Lancaster, Lancaster, LA1 4YL, U.K.
- Janson, S., Dept. of Math., Thunsbergstr. 3, 752 38 Uppsala, Sweden
- Jewell, N., Program in Biostatistics, School of Public Health, Univ. of California, Berkeley, CA 94720, U.S.A.
- Johnson, B.E., Dept. of Math., Univ. of Newcastle, Newcastle-upon-Tyne, NE1 7RU
- Kaashoek, M.A., Dept. of Math. and Computer Science, Vrije Univ., Amsterdam, Postbus 7167-1007 mc, Amsterdam, Netherlands
- Karanasios, S., Dept. of Math. National Technical Univ., 42, 28th October St., GR 106 82, Athens, Greece
- Katavolos, A., Dept. of Math., Univ. of Athens, Panepistimiopolis, Athens 15700, Greece
- Konig, H., Math Seminar, Univ. Kiel, 23 Kiel, W. Germany
- Korenblum, B., Dept. of Math. and Stat., SUNY, Albany, NY 12222, U.S.A.
- Lambrou, M., Dept. of Math., Univ. of Crete, P.O. Box 470, Crete, Greece
- Lerer, L., Math. Dept., Vrije Univ., 1007 MC, Amsterdam, Postbus 7161, The Netherlands
- Luecking, D., Dept. of Math. Sci., Univ. of Arkansas, Fayetteville, ARK 72701, U.S.A.
- Meinguet, J., Inst. Math., Univ. de Louvain, Chemin du Cyclotron 2, B-1348 Louvain-la-Neuve, Belgium
- Nikolskii, N.K., Dept. of Math., Steklov Inst., LOMI, Fontanka 27, Leningrad 191011, U.S.S.R.
- Øksendal, B., Dept. of Math. Univ. of Oslo, P.B. 1053, Blindern, Oslo 3, Norway
- Olin, B., Dept. of Math., Virginia Tech., Blacksburg, VA 24061, U.S.A.
- Peeters, G., Dept. of Math., Katholieke Univ. Leuven, Celestijnenlaan 200B, B-3030 Leuven, Belgium
- Peetre, J., Math. Inst., Box 725, S-220 07, Lund, Sweden
- Peller, V.V., Dept. of Math., Steklov Inst., LOMI, Fontanka 27, Leningrad 191011, U.S.S.R.
- Pestana, D., Dept. of Stat., Faculty of Sci., Univ. of Lisbon, 58 Rua da Escola Politénica, 1294, Lisbon, Codex, Portugal
- Power, S.C., Dept. of Math., Univ. of Lancaster, Lancaster LA1 4YL, U.K.
- Ran, A., Dept. of Math. and Computer Sci., Vrije Univ., Amsterdam, Postbus 7167-1007 mc, Amsterdam, Netherlands

- Rochberg, R., Dept. of Math., Washington Univ., St. Louis,
Missouri 63130, U.S.A.
- Rodman, L., Dept. of Math., Univ. of Calgary, 2500 Univ., Dr.,
Calgary, T2N 1N4, Canada
- Roozmond, L., Dept. of Math. and Computer Sci., Vrije Univ.,
Amsterdam, Postbus 7167-1007 mC, Amsterdam, Netherlands
- Rudin, W., Dept. of Math., Univ. of Wisconsin, 805 Van Vleck Hall,
Madison, WI 53706, U.S.A.
- Sarason, D., Dept. of Math., Univ. of California at Berkeley,
Berkeley, Ca 94720, U.S.A.
- Shapiro, H., Math. Inst., Royal Inst. of Technology, S-10044,
Stockholm, Sweden
- Shields, A., Dept. of Math., Univ. of Michigan, Ann Arbor, MI
48109-1003, U.S.A.
- Sinclair, A., Dept. of Math. Univ. of Edinburgh, Mayfield Rd.,
Edinburgh EH9 3JZ, U.K.
- Singh, S.P., Dept. of Math., Memorial Univ., St. John's, Newfound-
land, Canada A1C 5S7
- Seubert, S.M., Dept. of Math., Univ. of Virginia, Charlottesville,
VA 22903-3199, U.S.A.
- So, W.S., Dept. of Math., Univ. of Hong Kong, Hong Kong
- Sun Shunhua, Dept. of Math., Sichuan Univ., Sichuan, Peoples
Republic of China
- Timoney, R., Dept. of Math., 39 Trinity College, Dublin 2, Ireland
- Trent, T., Dept. of Math., Univ. of Alabama, AL 35486, U.S.A.
- Tuncay, H., Dept. of Math., Ege Univ., Izmir, Turkey
- Upmeyer, H., Math. Inst., Univ. of Tübingen, Auf der Morgenstelle
10, D-7400 Tübingen, W. Germany
- Waelbrueck, L., Dept. of Math., Univ. Libre de Bruxelles, Campus
Plaine LP 214, B-LU50, Brussels, Belgium
- Walsh, D., Dept. of Math., Maynooth College, Co. Kildare, Ireland
- Wogen, D., Dept. of Math., Univ. of North Carolina, Chapel Hill,
NC 27514, U.S.A.
- Young, N., Dept. of Math., Univ. of Glasgow, Glasgow, G12 8QW, U.K.
- Youngson, M., Dept. of Math., Heriot-Watt Univ., Edinburgh EH14
4AS, U.K.
- Younis, R., Dept. of Math., Kuwait Univ., P.O. Box 5969, Kuwait.

PROGRAMME OF THE INSTITUTE

16th July

Sarason	Operator theoretic aspects of the Nevanlinna-Pick interpolation problem I.
Conway	A survey of some results on subnormal operators I.
Nikolskii	New results and methods in the theory of Hankel and Toeplitz operators I.
Axler	Multipliers of the Dirichlet space.
Cowen	Composition operators on H^2 .
Jewell	Some open problems in function theory related to the characterisation of stochastic processes.
Fisher	Optimal estimation of analytic functions.
Wogen	Composition operators on H^2 of the ball.
Pestana	Functional analysis approach to limit distributions.
Sarason	Operator theoretic aspects of the Nevanlinna-Pick interpolation problem II.

17th July

Sarason	Operator theoretic aspects of the Nevanlinna-Pick interpolation problem III.
Conway	A survey of some results on subnormal operators II.
Nikolskii	New results and methods in the theory of Hankel and Toeplitz operators II.
Gorkin	Division in Douglas algebras.
Adams	Bergman bilateral shift.
Trent	Approximation by polynomials in $L^2(\mu)$.
Korenblum	Some spaces between BMO and Bloch.
Dudziak	Subnormal operators and weak star density.
Singh	On proximity maps.
Sarason	Operator theoretic aspects of the Nevanlinna-Pick interpolation IV.

18th July

- Sarason Operator theoretic aspects of the Nevanlinna-Pick interpolation V.
- Conway A survey of some results on subnormal operators III.
- Nikolskii New results and methods in the theory of Hankel and Toeplitz operators III.
- Rudin The modulus of H^2 -functions on T^n .
- Berg Compact approximations, Hankel operators and problems.
- Konig A trace theorem for operator polynomials.
- Gokturk Representation theorems for analytic functions with quasimeromorphic extensions.
- Olin Invariant subspaces for analytic functions of a subnormal operator.
- Meinguet On the Davis-Kahan-Weinberger solution of the norm preserving dilation problem.
- Shields Cyclic vectors in spaces of analytic functions I.

19th July

- Anderson Bloch space : the basic theory I.
- Conway A survey of some results on subnormal operators IV.
- Nikolskii New results and methods in the theory of Hankel and Toeplitz operators IV.
- Amar Extension and division of holomorphic functions of one or several variables.
- Clancey Operators with 1-dimensional self-commutator.
- Davidson Failure of the distance formula.
- Berger A strange function theory arising out of the creation operator.
- Sun Shunhua Halmos' 5th question and subnormal Toeplitz operators.
- Lambrou Rank one elements of reflexive algebras.
- Shields Cyclic vectors in spaces of analytic functions II.

20th July

- Anderson Bloch space : the basic theory II.
- Rochberg Decomposition theorems for Bergman spaces and their applications I.
- Peetre Paracommutators and minimal spaces I.
- Brown Cyclic vectors of bounded characteristic in Bergman spaces.

Peller	Some questions in the spectral theory of Toeplitz operators.
Luecking	Operators in Bergman Spaces.
Younis	Distance estimates and Toeplitz operators.
Dym	Unitary interpolants, factorisation indices and infinite Hankel block matrices.
Shields	Cyclic vectors in spaces of analytic functions III.
Peller	De Branges proof of the Bieberbach Conjecture.

21st July

Anderson	Bloch space : the basic theory III.
Rochberg	Decomposition theorems for Bergman spaces and their applications II.
Peetre	Paracommutators and minimal spaces II

23rd July

Young	Interpolation by analytic matrix functions I.
Rochberg	Decomposition theorems for Bergman spaces and their applications III.
Peetre	Paracommutators and minimal spaces III.
Haslinger	Weighted spaces on entire functions.
Walsh	Norm estimates for partial sum Hankel operators.
Francis	Uniformly optimal control of linear systems.
Bonsall	Decomposition of integrable functions as ℓ^1 sums of the positive kernel functions.
Peller	Applications of Hankel operators to the perturbation theory of self-adjoint and unitary operators.
Doyle	Engineering applications of matrix interpolation theory.
Helton	Optimisation, Engineering and a more general Corona Theorem I.

24th July

Young	Interpolation by analytic matrix functions II.
Øksendal	Stochastic Processes, Characteristic Operators and Function Theory I.
Kaashoek	Minimal factorisation, Linear systems and Integral operators I.
Rochberg	Decomposition theorems for Bergman spaces and their applications IV.
Ghahramani	Compact multipliers on weighted group algebras.
Lerer	Trace formulas for convolution-type operators.

- Upmeyer Toeplitz operators on bounded symmetric domains.
 Ball A matrix analogue of Hardy space function theory
 : factorisation, interpolation, dilation and
 lifting.
 Helton Optimisation, Engineering and a more general
 Corona Theorem II.

25th July

- Young Interpolation by analytic matrix functions III.
 Øksendal Stochastic Processes, Characteristic Operators
 and Function Theory II.
 Kaashoek Minimal factorisation, Linear systems and
 Integral operators II.
 Peetre Paracommutators and minimal spaces IV.
 Gohberg Integral equations and systems.
 Holland An extension of Milin's Tauberian theorem.
 Bart The coupling method for solving integral
 equations.
 Rodman Toeplitz operators with rational matrix symbols.
 Helton Optimisation, Engineering and a more general
 Corona Theorem III.

26th July

- Peetre Paracommutators and minimal spaces V.
 Øksendal Stochastic Processes, Characteristic Operators
 and Function Theory III.
 Kaashoek Minimal factorisation, Linear systems and
 Integral Operators III.