Autowave Processes in Kinetic Systems

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Autowave Processes in Kinetic Systems

**Spatial and Temporal Self-Organization in Physics,
Chemistry, Biology, and Medicine**

**V. A. Vasiliev, Yu. M. Romanovskii,
D. S. Chernavskii, V. G. Yakhno**

with 113 figures

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CIP

“... In the complex field of nonlinear oscillations, some specific general concepts, ideas, and methods will crystallize and be adopted for general use by the physicist to an even greater extent than is the case today. These concepts, ideas, and methods will become obvious and natural, and they will enable the physicist to analyze complicated sets of phenomena. They will become a powerful heuristic tool for new research.

The physicist interested in the modern problems of oscillations must, in my opinion, be taking this approach right now. He must understand the existing mathematical methods and approaches which are at the basis of these problems, and he must learn how to use them.”

L. I. Mandelshtam

(Preface to the book “Theory of Oscillations”, by A. A. Andronov, A. A. Witt, and S. E. Khaikin, published in 1935.)

Preface

*Probably, we are obliged to Science,
more than to any other field
of the human activity,
for the origin of our sense
that collective efforts are necessary indeed.*

F. Joliot-Curie

The study of autowave processes is a young science. Its basic concepts and methods are still in the process of formation, and the field of its applications to various domains of natural sciences is expanding continuously.

Spectacular examples of various autowave processes are observed experimentally in numerous laboratories of quite different orientations, dealing with investigations in physics, chemistry and biology. It is our opinion, however, that if a history of the discovery of autowaves will be written some day its author should surely mention three fundamental phenomena which were the sources of the domain in view. We mean combustion and phase transition waves, waves in chemical reactors where oxidation–reduction processes take place, and propagation of excitations in nerve fibres. The main tools of the theory of autowave processes are various methods used for investigating nonlinear discrete or distributed oscillating systems, the mathematical theory of nonlinear parabolic differential equations, and methods of the theory of finite automata. It is noteworthy that the theory of autowaves has been greatly contributed to be work of brilliant mathematicians who anticipated the experimental discoveries in their abstract studies. One should mention R. Fisher (1937), A. N. Kolmogorov, G. I. Petrovskii, and N. S. Piskunov (1937), N. Wiener and A. Rosenbluth (1946), A. Turing (1952).

Remarkably, these venerable mathematicians turned to problems in biology; the contemplation of mechanisms of life prompted them to state new ideas in mathematics. Their success in this field was due, in essence, to their understanding of two fundamental factors acting simultaneously in distributed active systems: autocatalysis and diffusion.

An important milestone in the progress of the theory of autowave processes was the invention of models for nervous conductivity, by A. Hodgkin and A. Huxley, and a model for the propagation of excitations in conducting system of heart muscle by D. Noble. These models — systems of partial differential equations — described not mental, but real experiments.

We have a right to claim that the Soviet school incorporating mathematicians, physicists, chemists and biologists is highly experienced in the study of autowave processes, its contribution to the subject is an essential one. So it seems fairly natural that the term itself, “autowave process” or “autowave”, was proposed by an outstanding Soviet physicist R. V. Khokhlov who possessed a deep understanding of the

role of the theory of nonlinear oscillations, not only in electronics and mechanics, but also in other sciences. Soviet scientists have contributed substantially, in particular, to the theories of combustion, formation of the universe structures, wave propagation and formation of strata in plasmas, chemical reactors, living tissues, and to the theory of signal transport in active lines.

During recent years, the role of the concept of autowave processes has become clearer in the development of synergetics, the study of open systems remote from thermodynamical equilibrium. It would be reasonable to assert that autowaves, besides the processes creating "new" information in such systems, are the mainsprings of self-organization. An understanding of this profound relation has been inspired to an appreciable extent by the work of I. Prigogine and H. Haken. Probably the most exhaustive exposition of the problems relevant to the history and recent developments of the study of autowave processes was presented at two International Symposia which took place at the Humboldt University, GDR (January, 1982) and in Pushchino, USSR (June, 1983). Nevertheless, a presentation of fundamental experimental facts and the main concepts of the theory of autowave processes has embarrassed the present authors in a number of points. Among the difficulties we met were the absence of a commonly adopted terminology, an uncertainty in determining the boundaries for the new field of science (as it is the case with any new trend originating at frontiers of different sciences), and the multitude of publications which are scattered in numerous books and a great number of various periodicals. Above all, the authors are physicists, so the exposition of mathematical aspects is presented at a "physical" level of rigor.

The authors emphasize that their book is essentially a collective work; each author is responsible for all chapter. The presentation of the Material would be impossible without preliminary discussions with many colleagues.

The authors want to express their profound gratitude for goodwill and encouragement and for discussion of a number of problems to S. A. Akhmanov, M. T. Grekhova, Yu. L. Klimontovich, E. V. Astashkina, H. Bartsch, S. I. Beylina, B. N. Belintsev, Yu. M. Galperin, Yu. A. Danilov, S. D. Drendel, G. G. Elenin, Yu. D. Kalafati, B. N. Klochkov, I. K. Kostin, V. I. Krinskii, S. P. Kurdyumov, K. K. Likharev, A. S. Logginov, I. I. Minakova, A. S. Mikhailov, G. A. Nepokoichitskii, M. S. Polyakova, A. V. Priezzhev, M. I. Rabinovich, S. A. Regirer, N. V. Stepanova, V. A. Teplov, I. V. Uporov, R. Feistel and others. The authors are extremely thankful to A. V. Kraev and L. Schimansky-Geier for their kind assistance in the translation of the book. We also greatly appreciate help rendered by L. M. Andrianova and N. I. Vasileva in arranging the manuscript.

Finally, we are very grateful to Professor Werner Ebeling of the Humboldt University Berlin who initiated the composition of the present book. Without his constant attention and support the text would never have been completed to the extent of the authors' ability.

Editor's Preface

Approach your problems from the right end and begin with the answers. Then one day, perhaps you will find the final question.

'The Hermit Clad in Crane Feathers' in
R. van Gulik's *The Chinese Maze Murders*.

It isn't that they can't see the solution.
It is that they can't see the problem.

G. K. Chesterton. *The Scandal of Father Brown* 'The point of a Pin'.

Growing specialization and diversification have brought a host of monographs and textbooks on increasingly specialized topics. However, the "tree" of knowledge of mathematics and related fields does not grow only by putting forth new branches. It also happens, quite often in fact, that branches which were thought to be completely disparate are suddenly seen to be related.

Further, the kind and level of sophistication of mathematics applied in various sciences has changed drastically in recent years: measure theory is used (non-trivially) in regional and theoretical economics; algebraic geometry interacts with physics; the Minkowsky lemma, coding theory and the structure of water meet one another in packing and covering theory; quantum fields, crystal defects and mathematical programming profit from homotopy theory; Lie algebras are relevant to filtering; and prediction and electrical engineering can use Stein spaces. And in addition to this there are such new emerging subdisciplines as "experimental mathematics", "CFD", "completely integrable systems", "chaos, synergetics and large-scale order", which are almost impossible to fit into the existing classification schemes. They draw upon widely different sections of mathematics. This programme, Mathematics and Its Applications, is devoted to new emerging (sub)disciplines and to such (new)interrelations as *exempla gratia*:

- a central concept which plays an important role in several different mathematical and/or scientific specialized areas;
- new applications of the results and ideas from one area of scientific endeavour into another;
- influences which the results, problems and concepts of one field of enquiry have and have had on the development of another.

The Mathematics and Its Applications programme tries to make available a careful selection of books which fit the philosophy outlined above. With such books, which

are stimulating rather than definitive, intriguing rather than encyclopaedic, we hope to contribute something towards better communication among the practitioners in diversified fields.

Because of the wealth of scholarly research being undertaken in the Soviet Union, Eastern Europe, and Japan, it was decided to devote special attention to the work emanating from these particular regions. Thus it was decided to start three regional series under the umbrella of the main MIA programme.

The phrase "Autowave processes" is perhaps not yet well and universally known. The subtitle of the book "Spatial and temporal self-organisation in physics, biology and medicine" is perhaps an easier understandable description of the subject of this book. Mathematically speaking we are dealing with evolution equations which have both autocatalytic and diffusion terms, as follows

$$\frac{\partial u_i}{\partial t} = f_i(u_1, \dots, u_m) - \sum_{l=1}^n \frac{\partial}{\partial x_l} (v_{il} u_i) + \sum_{l,k} \frac{\partial}{\partial x_l} \left(D_{ik} \frac{\partial u_k}{\partial x_l} \right)$$

They are also called (generalized) reaction-diffusion equations. Here $u_1, \dots, u_m$ are e.g. m substances, $f_i(u_1, \dots, u_m)$ is a "reaction term" which may embody autocatalytic phenomena (think of an equation like $y' = y^2$, $y(0) = c^{-1}$, $c \neq 0$, with the finite escape time solution $y(t) = (c - t)^{-1}$), v_{il} is the l -th component of the velocity of substance i , and the D_{ik} are diffusion coefficients which are positive functions of x, t and which often are assumed to be constant. The reaction terms $f_i(u)$ may give rise to steep gradients, shock waves, etc., a tendency which is balanced by the diffusion terms. As a result there may arise phenomena of pattern generation such as stationary spatially inhomogeneous stable solutions. In addition, such equations can exhibit localized wave sources, reverberation phenomena, travelling shape preserving solitary waves of a completely different kind than solitons, and dissipative structures. The equations often arise as mass balance equations

$$\frac{\partial u_i}{\partial t} = f_i(u) - \operatorname{div} I_i$$

where I_i is the flux of substance i , $I_i = v_i u_i \Sigma D_{ik} \operatorname{grad} u_k$, and they serve e.g. as equations describing nerve impulse transmission (Hodgkin-Huxley model), a phase transition wave, a chemical reaction process (e.g. the oscillating Belousov-Zhabotinskii reaction) combustion processes, and the evolution of a multispecies population with random migration. Quite generally they occur in connection with all kinds of processes in excitable media.

The term "autowave processes" for all these (and other) phenomena was coined by the USSR physicist R.V. Khokhlov. There are definite and important relations between these autowaves and ideas from synergetics and self-organisation. As indicated above, autowaves occur in medicine, physics, biology, chemistry; they also occur in mechanics and electronics. Thus the field of autowaves is not so easy to delimit and describe. Mathematically, we just have to study the equation type given above. Thus one way to view the field is a part of applied mathematics with lots of

applications in lots of different fields and with lots of experimental facts to guide intuition and to help one's thinking about these equations. A topic therefore which fits the underlying philosophy of the book series perfectly. I hope that — like me — many readers will find it an enormously stimulating introduction to the fascinating world of self-organisation and pattern formation.

The unreasonable effectiveness of mathematics in science ...

Eugene Wigner

Well, if you know of a better 'ole, go to it.

Bruce Bairnsfather

What is now proved was once only imagined.

William Blake

As long as algebra and geometry proceeded along separate paths, their advance was slow and their applications limited.

But when these sciences joined company they drew from each other fresh vitality and thenceforward marched on at a rapid pace towards perfection.

Joseph Louis Lagrange.

Bussum, October 1986

Michiel Hazewinkel

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