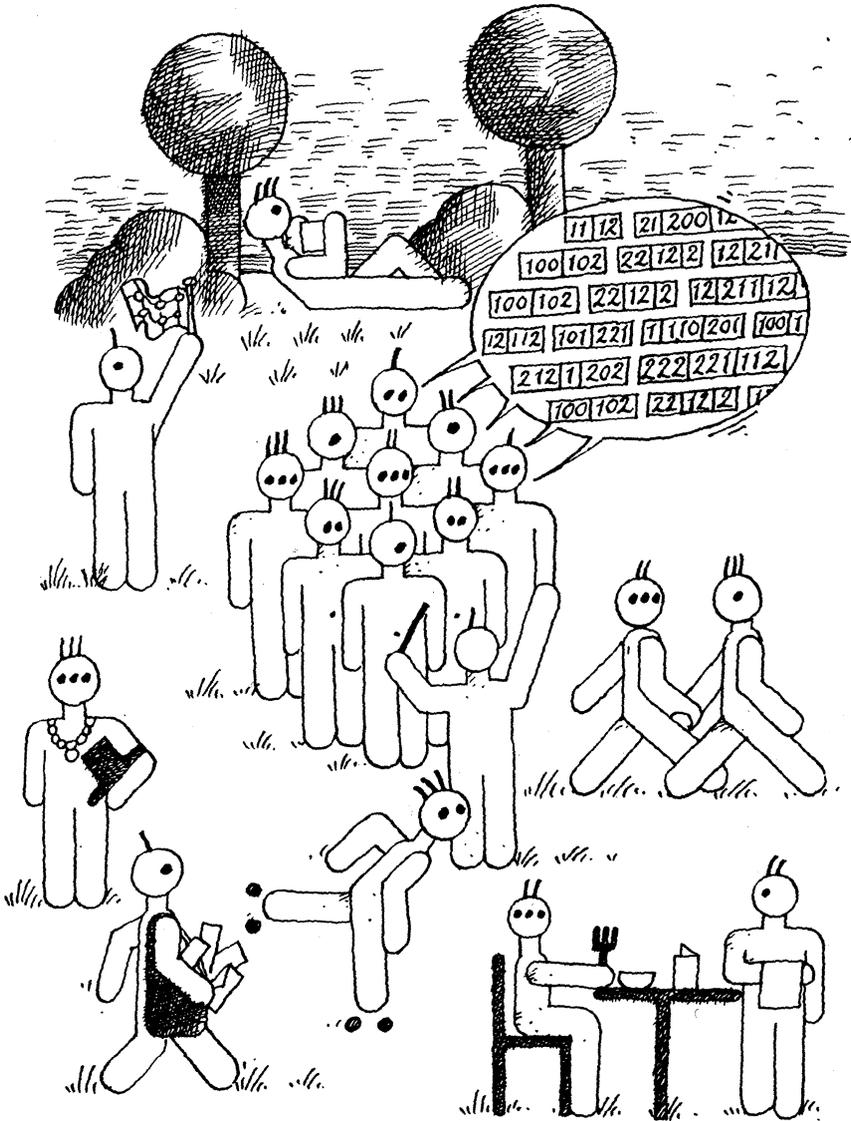


Mathematical Material for Chapter V



Freckleham

The “Freckleham” theme forms the basic material with which three-dimensional goal description as prepared by us will be demonstrated. The various assignments from the theme can be found in Chapter V together with the addition of a goal description in a didactical context.

This goal description will only be clearly understood by the reader if he himself has first solved the problems in “Freckleham”, working in a way that would be similar to that of the third grade pupils for whom the theme was designed.

FRECKLEHAM

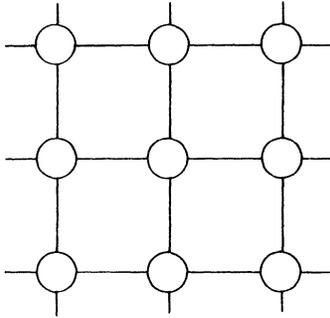
1. *The People of Freckleham*

The people of Freckleham are strange creatures. The mayor has three freckles and three hairs and the postman has one freckle and one hair. Every Frecklehammer is different from the other and has at least one freckle and one hair but no more than three freckles and three hairs.

Draw a picture of each of the Frecklehammers. Make sure you do not forget anyone.

2. *A Map of Freckleham*

Years ago the Frecklehammers chose where they wanted to live in their town, but at the postman's request the mayor changed all that.



Arrange the Frecklehammers in an orderly fashion on the grid above.

— Where could  live?*

— Where could  live?

— Give names to the streets and avenues.

* The black sections represent either a hat or a scarf and therefore conceal either hairs or freckles.

3. *Greetings*

Once they had been re-housed in a more orderly fashion and the postman

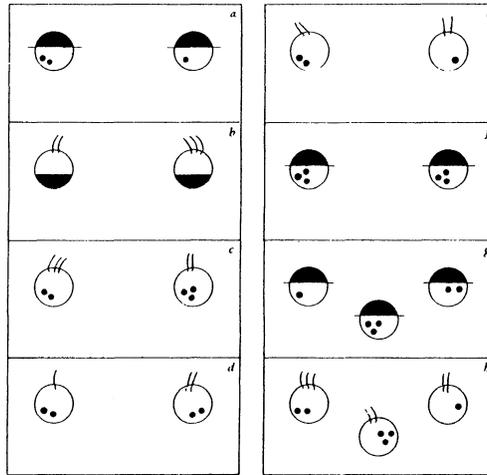
could work more efficiently, the mayor also decided to improve the manners of his townsfolk. He issued an order: when two Frecklehammers meet, the one with the most hairs or freckles will greet the other and say “I have more . . . than you have”.

“I  you” means: “I have more hairs than you have”.

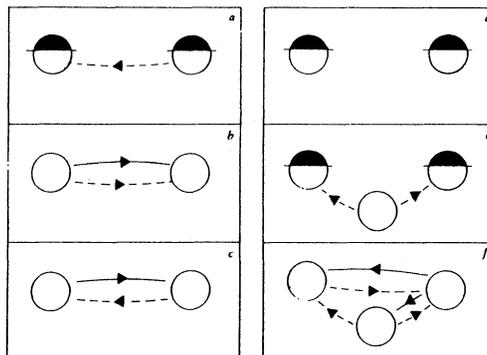
“I  you” means: “I have more freckles than you have”.

The townsfolk started to practise this and so will we:

— Draw in the arrows for the following pictures:

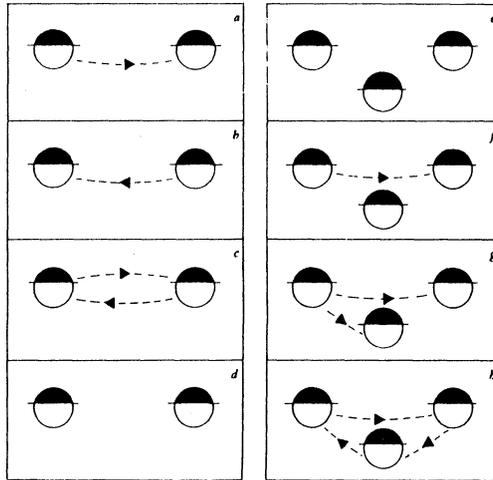


— Draw in hairs and/or freckles to fit the patterns below.



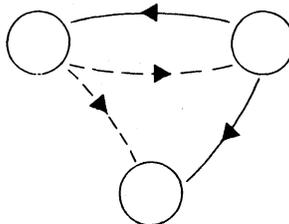
4. *Confusion*

— Which of the patterns can cause confusion or an argument? (No arrow means that no greeting takes place). Change the arrows when necessary (in red ink).



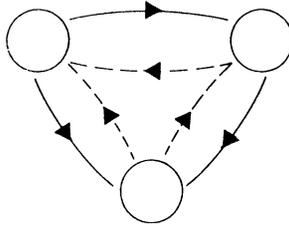
5. *Thieves*

Soon each Frecklehammer had learned to greet the others in the proper manner, but not every Frecklehammer was happy with the new situation. One day a policeman from a neighbouring town came to see the mayor. He reported that three Frecklehammers had tried to break into the barber's shop in his town. They were looking for more hairs and freckles! The three thieves had escaped. The policeman did not know what they looked like, but he said he had heard their greetings. He drew the sketch show below:



— What could the three thieves have looked like?

But the policeman had not heard correctly. His fellow officer remembered the greeting exactly. That made it easier to identify the thieves.



— Who are the thieves?

6. *The Town Meeting*

Somehow the people of Freckleham seemed less contented than before. The Mayor had become aware of this since the robbery. He decided to call a meeting of all the townspeople to decide what could be done to improve matters.

— The greeting “I have more . . . than you have” was heard many times that night. How many times?

7. *New Proposals*

It soon became clear that many of the Frecklehammers were unhappy with the current way of greeting each other. It sounded so boastful and the postman especially resented the fact that he had to stay quiet during all greetings. He reasoned that everyone should be able to say hello and he proposed to do so by saying “I have as many . . . as you have”. The emphasis would lie on the similarities and not on the differences. Everyone was pleased by the new idea. They began to practise, and so will we.

— Do you think that everyone will be satisfied by this new way of greeting each other?

Discuss these questions:

- How many greetings will be heard at the next town meeting?
- How many times will each person greet another at the next meeting?

8. *The Freckleham Song*

From that time the Frecklehammers celebrated their new way of greeting on an “as many as” day. The Freckleham song was composed to be sung on that day and the words were engraved on the wall of the town hall. It looked like this:

100 22,1,211,12 1,201 111,1,112,221 22,1,100,200,201
 1,201 221,120,210.
 100 22,1,211,12 202,22,1,202 111,1,112,221
 20,200,12,10,102,110,12,201 202,120,120.
 1,112,11 221,120,210 11,120 1,201 100 11,120
 100 22,1,211,12 202,22,1,202 111,1,112,221 202,120,120.
 100 22,1,211,12 1,201 111,1,112,221 1,201 221,120,210.

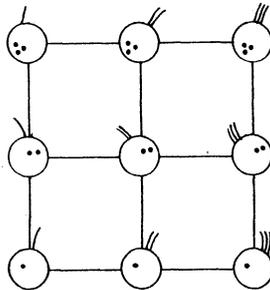
In Freckleham only three digits 0, 1, 2 are used for everything including reading and writing. Strange! In the first grade of school little Frecklehammers learn that $1 + 1 = 2$ and $2 + 1 = 10$. Imagine that!

I have not been able to decode the Freckleham Song, which consists of exactly ninety (which is 10100 in Freckleham) “letters”. Perhaps no one will ever be able to discover the words to the song and it will remain a secret for ever. I do know that the Frecklehammers lived happily ever afterwards. The fact that the mayor’s new form of greeting had to be altered only goes to prove that “I have more than you have” does not always make people happy.

— Try to find the words to the Freckleham Song.

ANSWERS

1. There are 9 Frecklehammers (see the choir on the title page).
2. One possibility: the Frecklehammers with 1 hair in the left-hand column, with 2 hairs in the middle column and with 3 hairs in the right-hand column; one freckle bottom row, 2 freckles middle row, 3 freckles top row. This results in the following pattern:



Of course a two-way arrangement can be displayed differently, and there are 8 possibilities.

3. N.B. The absence of an arrow means that the characteristics are alike.
4. *c* will cause a problem since one or both of the two Frecklehammers will greet each other incorrectly.

f contains a mistake: the lower Frecklehammer has just as many freckles as the other two, who do not have the same number.

h cannot be correct either, since each of the three Frecklehammers would have more than the others.

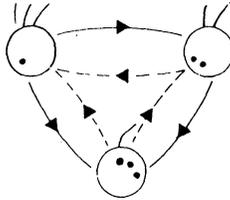
5. There are several possibilities for the first group of three Frecklehammers. Looking only at the hairs we have:

(2,1,1)

(3,1,1)

(3,2,2)

There are also 3 possibilities for the freckles. A combination of the two results in 9 groups of three Frecklehammers. There is only one possible answer for the second pattern:



The quickest way of finding the answer is to consider the hair and freckle forms of greeting separately. The Frecklehammer that greets twice has 3 hairs (freckles) and the one that greets once has 2 hairs (freckles). The one that says nothing has 1 hair (freckle). This conclusion can be drawn since there are no 'equals' present, i.e., no arrows missing.

6. The total number of greetings can be found by using the grid. Consider the hairs first. Each Frecklehammer greets his lesser fellow, and therefore his left-hand neighbour. This means that those with 3 hairs greet $3 \times 6 = 18$ times and those with 2 hairs $3 \times 3 = 9$ times. All together 27 times. The same is true for the freckles. In the grid this is made visible by "going down". A total of 54 greetings is obtained.
7. Each Frecklehammer greets all the others in his street or avenue thus resulting in four times. The total is $4 \times 9 = 36$.
8. The letters are coded according to their position in the alphabet written in the base three system. The words to the song are therefore:

I have as many hairs as you.
 I have that many freckles too.
 And you do as I do,
 I have that many too.
 I have as many as you.