Mathematical Material for Chapter IV

The Land of Eight

"The Land of Eight" theme is meant for the fifth grade of the elementary school. It serves here to illustrate the problem area of goal description for Chapter IV.

The leading question is how to describe the objectives of this theme for the "less-familiar" reader. In order to focus as sharply as possible on goals and goal description, we have not filled out the concluding part of the theme, which has an evaluative character. The reader is asked to draw his own final conclusions and to compare his solutions with the two examples in the text of Chapter IV. Thus different points of view about mathematics education that are possible can emerge clearly.

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THE LAND OF EIGHT

In this theme the pupils are taken to a country with strange arithmetical rules. By taking a closer look at these rules, the children are offered the opportunity to "back-track" on what they have already learnt about basic operations in order to re-discover the properties of the decimal system. At the same time they learn to see that the particular base of a positional notation is irelevant for the purposes of counting and calculating.

1. *Introduction to "The Land of Eight"*

In the first lesson the pupils are introduced to "The Land of Eight". After an introductory discussion about Walt Disney cartoons, the first work sheet is handed out. The children are asked whether they notice any peculiarities. After a number of comments, one pupil discovers that each of the characters has only four fingers on each hand.

They are told that Disney did this on purpose: it saved time. Thousands of drawings are necessary for one cartoon. And what is more, most people never noticed that something was "missing" in the cartoon. They did notice this when the figures had three instead of five fingers. Therefore further thrift was unwarranted.

Up to this point the cartoon characters were doing well, but once they were animated in the cartoon — which is nothing more than a series of pictures — their problems started. By the use of story-like characters the children realise that we have entered into the fantasy world of motion pictures.

What were these problems? The cartoon characters, and especially the children among these characters, were having trouble with counting and calculating. Why was that?

One pupil suggested that it had something to do with counting on one's fingers. We follow this suggestion: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. We analyse what the symbol "ten" (10) really means: I've used each of my hands once in counting, we call it a *grasp*, and now I start counting again. Twenty-three (23) means: both hands twice and three more.

The cartoon children could not count in this fashion, so it was decided to count in a different way. To make clear what they had in mind, they talked about "The Land of Eight."

2. *Counting*

First: how do cartoon parents teach their children to count? How many digits will they need?

Some answer, seven; others say, eight. We still cannot be sure what their answers mean. They write down their answers and the following solutions are offered:

1, 2, 3, 4, 5, 6, 7
0, 1, 2, 3, 4, 5, 6, 7
1, 2, 3, 4, 5, 6, 7, 8
0, 1, 2, 3, 4, 5, 6, 7, 8
1, 2, 3, 4, 5, 6, 7, 10.

The last suggestion might be chosen, but let us now assume not. It would be best to consider the row 0 through 7 or 0 through 8 and analyze it analogously to the decimal sequence. We see that for eight we must say: "Both hands once" (a grasp) and write down "10" but now pronouncing it as one, zero — this is to avoid confusion with the decimal system. In "The Land of Eight", "nine" becomes "11" being both hands once plus one, and is pronounced as one, one. (From now on in the text base eight will be given between quotation marks).

3. *Tallying*

How do we tally in fives? Why do we do it like this? How would they tally in "The Land of Eight". The pupils are asked to tally quantities:

𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 |||

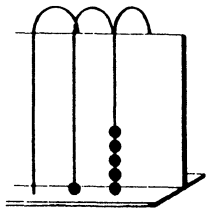
The idea is that the pupils should find a good counting strategy: two groups of four make "10" (one, zero), and after that it is easy to count: "10", "20", "30", "40", "47". It is more sensible to count directly using the base eight system rather than first using the decimal system and then having to convert the outcome into base eight:

$9 \times 4 + 3 = 39 =$ "47" (both hands four times plus seven). Both strategies — direct and indirect — are discussed.

4. *The Abacus*

The loop-abacus is demonstrated. Attention is directed to

— notation,
— carrying,
— use of the abacus in base ten and base eight:



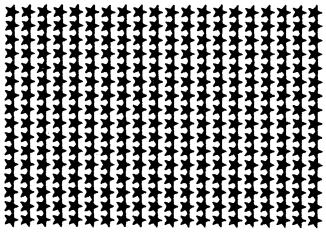
The assignment:

Write down in "The Land of Eight": 28, 43, 54, 64 (use the abacus if you like). Some of the pupils work with eights: 43 is five eight plus three = "53". Others work in groups with the aid of the abacus. The conversion of 64 fails for most at the first attempt. Answer: 64 = "80". They forget to change the eight eights.

The problem is discussed in groups. The abacus is used to demonstrate what happens. We remind the pupils of the odometer. They soon discover what they have forgotten: 64 = "80" = "100".

5. *Clever Counting*

The pupils are told to count the stars in the following pictures:



The problem can be solved in various ways:

- by counting: "1", "2", "3", "4", "5", "6", "7", "10", "11", . . .
- by conversion: 20 × 16 = 320, and attempting to write this down in base eight, which causes difficulties,
- by counting columns: "20" per column; the total being "20", "40", "60", "100", . . . "500".
- by making squares of eight by eight and then by counting in larger groups: "100", "200", "300", "400", "500".

The four methods and combinations of them are discussed and the additional method of multiplication is also suggested. Indeed, as the use of multiplication is the easiest way to reach the solution in the decimal system, could it also be used in base eight?

How can "20" × "24" be solved?

"2" × "24" = "48" = "50", therefore "20" × "24" = "500".

Are there any pupils who make suggestions in this direction?

The problem is left at this stage. The objective is to give the children an indication that it is possible to calculate in base eight.

6. *Addition and Subtraction*

The following problems are offered:

'7'	'3'	'567'	'100'	'10000'	'1245'	'1233'	'2350'
'6' +	'5' +	'765' +	'3' −	'100' −	'533' +	'5' −	'105' −

Addition causes few problems, while subtraction is much more difficult. Many pupils use the abacus in the latter case. We point out the analogy with the procedure used in the decimal system:

(a)	*(b)*	*(c)*
9 10 '100' '3' − 97	'7'8 '100' '3' − '75'	'7"10' '100' '3' − '75'

For (b) the two systems are mixed. This method makes it somewhat easier since the eight-fold appears as the familiar eight and not as the "10".

7. *Conclusion*

The two final lessons serve to evaluate and reflect on the previous ones. As mentioned earlier, the idea is to get the pupils to review what has been learned in order to make them aware of the arithmetical processes that are followed when using the decimal system and to make them realise that it makes no difference which system is used (see pp. 141—146).