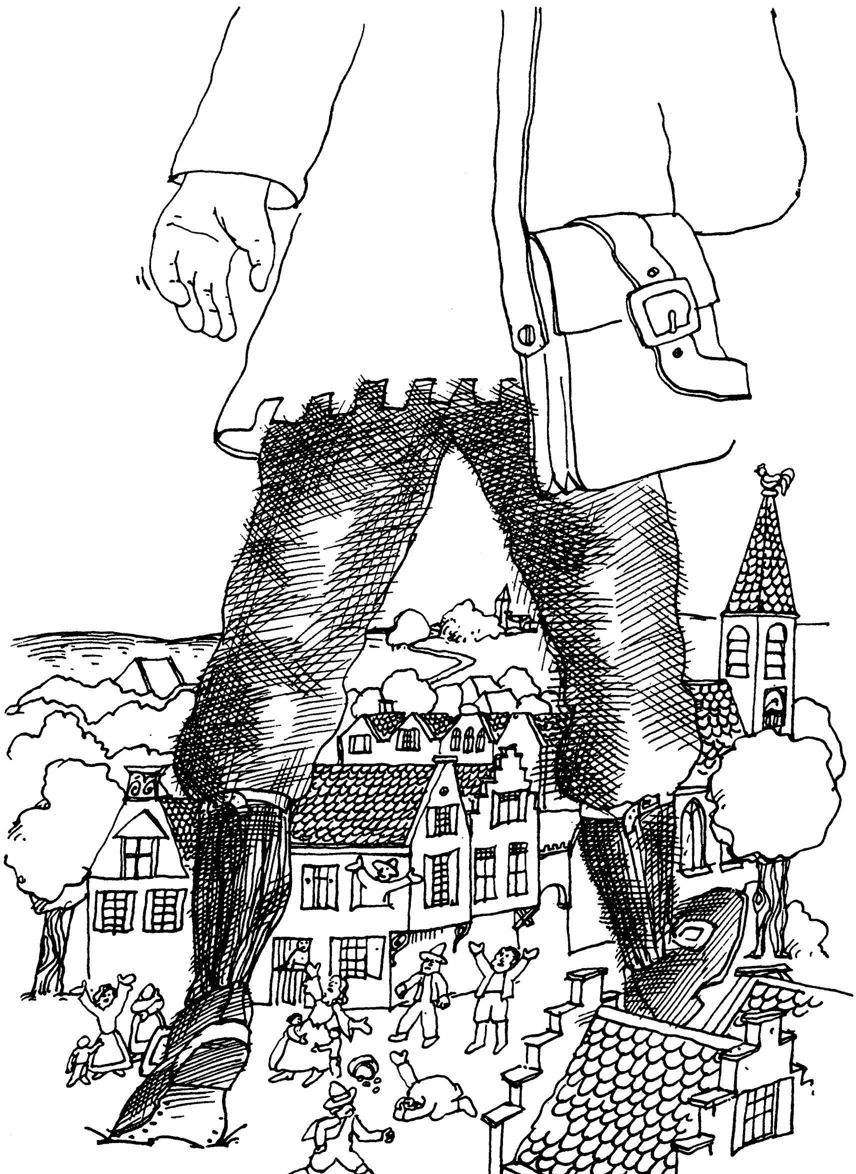
Mathematical Material for Chapter I



Gulliver

The topic "Gulliver" was designed for the sixth grade. Here it precedes the introductory chapter, and is meant to give a first impression of the kind of mathematics education Wiskobas supports. This first impression is brought out in full in the basic text of Chapter I by setting Wiskobas' ideas in the context of four trends in arithmetic/mathematics education. "Gulliver" also serves as a means of explaining the central problem.

The reader is asked to read the following text, to consider the mathematical problems it contains, and to focus on the background ideas of mathematics education that lie behind it, before reading Chapter I.

Designed by L. Streefland and A. Treffers

GULLIVER

The "Gulliver" theme considers how linear enlargement influences circumference, area and volume. The activities include

- — drawing a ground plan;
- — comparing objects from Lilliput with similar ones in our own world;
- — scrutinizing the credibility of certain quantitative data in "Gulliver's Travels";
- — analysing the concept of population density;
- — viewing the possibility of Lilliputians biologically;
- — examining a series of practical applications.

1. *Story*

In the first lesson the teacher reads from '*Gulliver's Travels*' by Jonathan Swift.

Could this story have really happened?

There is disagreement among the children. The arguments for yes and no are markedly different. Some of the children come forward with "hard" facts like a TV series about the Cyclops and excavations that confirm the existence of giants. Others give their personal opinions. The first lesson is a good starting point for scrutinizing a large amount of quantitative data in the story. It is also possible to dwell on the social background of the story and the intentions of the author.

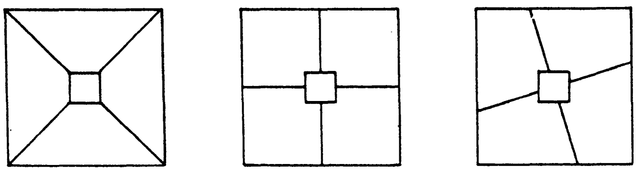
2. *Ground Plan*

The assignment in the following lesson consists of drawing a ground plan of Mildendo, the capital of Lilliput, by using the information from the following data.

The City is an exact Square, each side of the Wall being five hundred foot long. The two great Streets, which run across and divide it into four Quarters, are five foot wide. The Lanes and Alleys, which I could not enter, but only view them as I passed, are from twelve to eighteen Inches. The Town is capable of holding five hundred thousand Souls. The Houses are from three to five Stories. The Shops and Markets are well provided.

The Emperor's Palace is in the Center of the City, where the two great Streets meet. It is enclosed by a Wall two foot high, and twenty foot distant from the Buildings. I had his Majesty's Permission to step over this Wall: and the Space being so wide between that and the Palace, I could easily view it on every side. The outward Court is a Square of forty foot, and includes two other Courts: In the inmost are the Royal Apartments, which I was very desirous to see, but found it extremely difficult; for the great Gates, from one Square into another, were but eighteen Inches high and seven Inches wide. (p. 72–3 Swift, 1890)

Though this fragment gives some hints, there are a few other things which we need to know in order to make a detailed sketch. The main streets can be drawn in various ways:



The palace square is not unambiguous and various ideas about the scale are possible. All sorts of matters can lead to discussion.

3. *Population Density*

Now Mildendo is compared to a city from our world. The story has told us that the reduction factor is 12. All sorts of questions arise:

- — Are the main streets comparatively wide?
- — Aren't the smaller streets rather narrow?
- — What about the wall?
- — Can it be compared with the great wall of China?
- — How wide are our own streets and lanes?

In order to keep the comparison simple we must translate all measures either to the Lilliputian or to our own world.

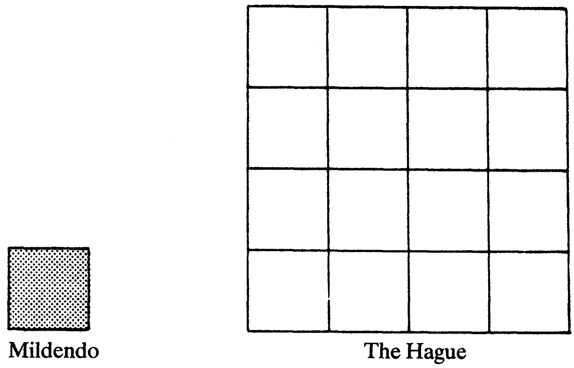
— Which shall we choose?

After some discussion it seems to be easier to choose our own measures. In terms of our measures "Mildendo" is only 1.8. by 1.8. or about 2 by 2 kilometres. But it has 500,000 inhabitants.

— How is that possible?

Some of the children look for a 2 by 2 km city in order to compare its population to that of Mildendo. They do not find any, so others suggest The Hague, which has about 500,000 inhabitants. They suggest comparing the area of The Hague which is referred to as a square of about 8 by

8 km. The children calculated it from the map. Who can explain the meaning of saying the population in Mildendo is very "dense"?



The pupils come forward with all sorts of arguments, but it is hard for them to explain something their intuition tells them is true, namely that 500,000 is a lot of people in such a small area. We give them a hint.

Suppose that The Hague was as densely populated.

— How many inhabitants would it have?

Now it is clear: more than half of the Dutch population live in The Hague.

But be careful! We should more properly have focused on a situation around 1750 (also for the width of the streets). At that time a city like Utrecht had a density of about 30,000 people per square kilometre. Swift did not exaggerate as much as the children had thought in the first instance.

4. *Gulliver's Clothes*

— Suppose we were Lilliputians, how big would Gulliver be?

— How big would his shoe, comb and handkerchief be?

The question about the handkerchief is especially important for the sequel.

— How many Lilliputian handkerchieves make one for Gulliver?

This question introduces the influence of linear enlargement on area. The pupils have no difficulty with the problem. Now we turn to the part of the story that deals with Gulliver's clothes:

Two hundred Sempstresses were employed to make me Shirts, and Linnen for my Bed and Table, all of the strongest and coarsest kind they could get, which, however, they were forced to quilt together in several Folds, for the thickest was some degrees finer than Lawn. Their Linnen is usually three Inches wide, and three foot make a Piece. The Sempstresses took my Measure as I lay on the ground, one standing at my Neck, and another at my Mid-Leg, with a strong Cord extended, that each held by the end, while the third measured the length of the Cord with a Rule of an Inch long. Then they measured my right Thumb, and desired no more; for by a mathematical Computation, that twice round the Thumb is once round the Wrist, and so on to the Neck and the Waist, and by the help of my old Shirt, which I displayed on the Ground before them for a Pattern, they fitted me exactly. (Swift, 1890, p. 90–91)

First of all we decide whether the 1 to 2 ratio between thumb and wrist, wrist and neck, neck and waist are correct. We start by estimating, which does not work, and so we measure. Then, we have children calculate how much material is needed and ask them how many Lilliputian clothes can be made from Gulliver's clothes. This problem tests whether or not the pupils see the connection with the handkerchief problem. But even more important is the link with the solution of the food problem.

5. *The Food Problem*

Intuitively one would assume that there is a linear relation between food and weight. How many Lilliputian portions does Gulliver eat? This leads us to the question of his weight. How many "Lilliputians" does Gulliver weigh? The question can be solved empirically in the first instance. We take a piece of clay or wood and compare its weight to a similar piece that is linearly three times as large. Then we prove that the weight of the large piece is 27 times that of the small piece. Gulliver weighs $12 \times 12 \times 12$ Lilliputians. We take it that he eats about 1728 Lilliputian portions per day. We read further in the book:

The Reader may please to observe, that in the last Article for the Recovery of my Liberty, the Emperor stipulates to allow me a Quantity of Meat and Drink sufficient for the Support of 1724 Lilliputians. Some time after, asking a Friend at Court how they came to fix on that determinate Number, he told me, that his Majesty's Mathematicians, having taken the Height of my Body by the help of a Quadrant, and finding it to exceed theirs in the Proportion of Twelve to One, they concluded from the Similarity of their Bodies, that mine must contain at least 1724 of theirs, and consequently would require as much Food as was necessary to support that number of Lilliputians. By which, the Reader may conceive an Idea of the Ingenuity of that People, as well as the prudent and exact Oeconomy of so great a Prince. (Swift, 1890, p. 71)

Without bothering about the error (1724 should be 1728) the calculation is found to be correct by our own mathematical wizards. Yet it is not correct!

Without further explanation, we supply the information that the amount of food is not proportional to the volume (weight) but to the surface area of the body. For the time being the biological explanation is omitted.

The consequences, however, must be faced. We can lead the train of thought as follows: One Gulliver balances with 1728 Lilliputians. In order to stay in balance and alive they must eat. One portion for Gulliver is one portion for each Lilliputian, as long as we assume that they eat the same amount in proportion. Now it has appeared that the amount of food depends on the surface area (remember the city, handkerchief, clothes).

That means that Gulliver's portion only feeds 144 (12 × 12) Lilliputians. That is 12 times as small. Therefore, proportionately, the Lilliputians must eat 12 times as much as a human. If they ever existed, they would need to eat all day. (See Note 16 for Chapter 1).

The mystery of Lilliput is unravelled without taking away any of the story's fascination. The effect of linear enlargement on higher dimensions can be experienced as a miracle of equal magnitude.

The theme is concluded by a number of test questions that relate to the heart of the problem.

One example: When Gulliver gets up out of the bath tub a thin layer of water covers his body. If it could be collected, it would fill a tall glass, the weight of which is about 1% of Gulliver's weight (1% of 70 kg.). Gulliver hardly notices the water.

But the Lilliputian who is 12 times smaller is bothered by the weight of the water on his skin. Try to explain this.