FRACTURE MECHANICS CRITERIA AND APPLICATIONS

ENGINEERING APPLICATION OF FRACTURE MECHANICS

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Fracture Mechanics Criteria and Applications

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Editor's preface

It is difficult to do justice to fracture mechanics in a textbook, for the subject encompasses so many disciplines. A general survey of the field would serve no purpose other than give a collection of references. The present book by Professor E. E. Gdoutos is refreshing because it does not fall into the esoteric tradition of outlining equations and results. Basic ideas and underlying principles are clearly explained as to how they are used in application. The presentations are concise and each topic can be understood by advanced undergraduates in material science and continuum mechanics. The book is highly recommended not only as a text in fracture mechanics but also as a reference to those interested in the general aspects of failure analysis.

In addition to providing an in-depth review of the analytical methods for evaluating the fundamental quantities used in linear elastic fracture mechanics, various criteria are discussed reflecting their limitations and applications. Particular emphases are given to predicting crack initiation, subcritical growth and the onset of rapid fracture from a single criterion. Those models in which it is assumed that the crack extends from tip to tip rely on the *specific* surface energy concept. The differences in the global and energy states before and after crack extension were associated with the energy required to create a unit area of crack surface. Applications were limited by the requirement of self-similar crack growth. Quantities such as energy release rate, stress intensity factor, path independent integral, etc. fall into this category. An alternative view based on physical observation is that crack growth is not continuous but is a discrete process of failure initiation, repeated many times, at a finite distance from the crack tip. Attention would then be focused on the energy stored in a unit volume of material reaching a critical state. Crack growth is considered as the loci of fracture initiation sites. Consistency can thus be achieved not only in assessing initiation, slow growth and rapid fracture but also in the simultaneous description of yielding and fracture. Macroplasticity off to the side of a macrocrack extension is distinguished from the microplasticity that may prevail ahead of a growing macrocrack. By the same token, microcracks may occur in regions of macroplasticity. The time and location at which energy is dissipated to deform and fracture at a specified size scale of material need to be identified. The book

gives a comprehensive explanation of how the stationary values of the strain energy density function can be applied to locate the sites of yield and fracture initiation. Yielding and fracture at the macroscopic level are shown to occur at different locations. The former always precedes the latter. Many illustrative examples are presented and critiqued against the other criteria. Fatigue crack growth can be treated in the same way by considering the cumulation of the strain on volume energy density. When the disturbance becomes time dependent, inertia effects of material elements around the crack come into play. This can occur regardless of whether the crack is stationary or moving. Further material on dynamic fracture can be found in the references.

The important contribution on applying fracture mechanics principles is not the number of examples but the methodology for resolving the loading rate and specimen size effect. Straight line relations obtained from the constant rate change of strain energy density factor with crack growth permit linear interpolation of the results. Data on crack growth can thus be obtained with a limited number of experiments and can be used to predict situations other than those tested. A realistic application of fracture mechanics could not be made without a sound understanding of the fundamentals. To this end, the book has met the objective.

*Bethlehem, Pennsylvania
December, 1989*

G. C. SIH

Preface

The objective of engineering design is the determination of the geometry and dimensions of machine or structural elements and the selection of material in such a way that the elements perform their operating function in an efficient, safe and economic manner. For this reason the results of stress and displacement analysis are coupled with an appropriate failure criterion, which is basically a postulate predicting the event of failure itself. Traditional failure criteria cannot adequately explain the number of structural failures that occur at stress levels considerably lower than the ultimate strength of the material. Example problems include bridges, tanks, pipes, weapons, ships, railways and aerospace structures. On the other hand, experiments performed by Griffith in 1921 on glass fibers led to the conclusion that the strength of real materials is much smaller, typically by two orders of magnitude, than their theoretical strength. In an effort to explain these phenomena the discipline of fracture mechanics has been created. It is based on the realistic assumption that all materials contain crack-like defects which constitute the nuclei of failure initiation.

Defects can appear in a structure in three major different ways: first, they can exist in a material due to its composition, as second-phase particles, debonds in composites, etc.; second, they can be introduced into a structure during fabrication, as in welds; and, third, they can be created during the service life of a component, like fatigue, environment-assisted or creep cracks. Fracture mechanics studies the load-bearing capacity of structures in the presence of initial defects, where a dominant crack is usually assumed to exist.

A new design philosophy is therefore introduced by fracture mechanics as opposed to the use of the conventional failure criteria. As catastrophic fracture is a consequence of the unstable propagation of a crack from a pre-existing defect, we are faced with the question: 'Can fracture be prevented by constructing structures that have no defects?' The answer is 'no', on the grounds of practicality. Then, the safe design of structures should proceed along two lines: either the *safe operating load* should be determined when a crack of a prescribed size is assumed to exist in the structure; or, given the operating load, the *size of the crack that is created in the structure* should be determined.

Fracture mechanics is searching for parameters which characterize the propen-

sity of a crack to extend. Such a parameter should be able to relate laboratory test results to structural performance, so that the response of a structure with cracks can be predicted from laboratory test data. This is determined as a function of material behavior, crack size, structural geometry and loading conditions. On the other hand, the critical value of this parameter – known as *fracture toughness*, a property of the material – is determined from laboratory tests. Fracture toughness expresses the ability of the material to resist fracture in the presence of cracks. By equating this parameter to its critical value a relation is obtained between applied load, crack and structure geometry which gives the necessary information for structural design. Fracture toughness is used to rank a material's ability to resist fracture within the framework of fracture mechanics, in the same way that yield or ultimate strength is used to rank a material's resistance to yield or fracture in the conventional design criteria. In selecting materials for structural applications a choice has to be made between materials with a high yield strength but comparatively low fracture toughness or a lower yield strength but higher fracture toughness.

The phenomenon of fracture of a solid is complicated and depends on a wide variety of factors, including the macroscopic effects, the microscopic phenomena which take place at the locations where the fracture nucleates or grows, and the composition of the material. The study of the fracture process depends on the scale level at which it is considered. At one extreme is the rupture of cohesive bonds in the solid, and the associated phenomena take place within distances of the order of 10^{-7} cm. For such studies the principles of quantum mechanics should be used. At the other extreme the material is considered as a homogeneous continuum and the phenomenon of fracture is studied within the framework of continuum mechanics and classical thermodynamics. Fracture studies that take place at scale levels between these two extremes concern movement of dislocations, formation of subgrain boundary precipitates and slip bands, grain inclusions and voids. Thus, the understanding of the phenomenon of fracture depends to a large extent on the successful integration of continuum mechanics with materials science, metallurgy, physics and chemistry. Due to the insurmountable difficulties encountered in an interdisciplinary approach the phenomenon of fracture is usually studied within only one of the three scale levels: namely, the atomic, the microscopic and the continuum. Attempts are under way to find a unified, interdisciplinary approach to the phenomenon of the failure of solids.

The purpose of this book is to present a clear, straightforward and unified interpretation of the basic problems of fracture mechanics with particular emphasis given to fracture mechanics criteria and their application in engineering design. The book is divided into nine chapters.

The first, introductory, chapter gives a brief account of some characteristic failures that could not be explained by the traditional failure criteria, and of Griffith's experiments which gave impetus to the development of a new philosophy in engineering design based on the discipline of fracture mechanics. The next two chapters deal with the determination of the stress and deformation fields in cracked bodies and provide the necessary prerequisite for the develop-

ment of the criteria of fracture mechanics. More specifically, Chapter 2 covers the basic analytical, numerical and experimental methods for determining the linear elastic stress field in cracked bodies, with particular emphasis on the local behavior around the crack tip, and Chapter 3 is devoted to the determination of the elastic-plastic stress and displacement distribution around cracks for time-independent plasticity. Addressed in the fourth chapter is the theory of crack growth, based on the global energy balance of the entire system. The fifth chapter deals with the theoretical foundation of the path-independent J -integral and its use as a fracture criterion. Furthermore, a brief presentation of the crack opening displacement fracture criterion is given. Chapter 6 studies the underlying principles of the strain energy density theory and demonstrates its usefulness and versatility in solving a host of two- and three-dimensional problems of mixed-mode crack growth in brittle and ductile fracture. Chapter 7 presents in a concise form the basic concepts and the salient points of dynamic fracture mechanics. Addressed in Chapter 8 is the phenomenon of fatigue and environment-assisted crack growth which takes place within the framework of the macroscopic scale level. Finally, Chapter 9 presents the basic principles of engineering design based on the discipline of fracture mechanics and gives a number of example problems. The applicability of fracture mechanics to composites and concrete is also discussed. The chapter concludes with a brief description of the more widely used nondestructive testing methods for defect detection.

Particular care was taken throughout the book to give a clear, consistent, simple and straightforward presentation of the basic concepts of the discipline of fracture mechanics from a continuum mechanics viewpoint. The book is self-contained and can be used as a textbook in undergraduate and postgraduate courses and as a reference book by all those who are interested in developing design methodologies and promoting research to include the influence of initial defects or cracks.

The author wishes to express his gratitude to Professor G. C. Sih of Lehigh University for his pioneering work on fracture mechanics on which parts of the book were based, his very stimulating discussions and his comments and suggestions during the writing of the book. Thanks are also due to my secretary, Mrs L. Adamidou, for typing the manuscript. Finally, I wish to express my gratitude to my wife, Maria, not only for proofreading the manuscript, but for her understanding and patience during the writing of the book.

Xanthi, Greece, 1989

EMMANUEL E. GDOUTOS