Algebraic Structures and Operator Calculus

Mathematics and Its Applications

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Volume 347

Algebraic Structures and Operator Calculus

Volume III: Representations of Lie Groups

by

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The MAPLE programming code pertaining to this book is available by anonymous ftp from: ftp.wkap.nl or by using your Web browser from URL: ftp://ftp.wkap.nl/software/algebraic_structures/lie-mapl.zip.

KLUWER ACADEMIC PUBLISHERS
DORDRECHT / BOSTON / LONDON

A C.I.P. Catalogue record for this book is available from the Library of Congress.

ISBN-13: 978-94-010-6557-3 e-ISBN-13: 978-94-009-0157-5
DOI: 10.1007/978-94-009-0157-5

Published by Kluwer Academic Publishers,
P.O. Box 17, 3300 AA Dordrecht, The Netherlands.

Kluwer Academic Publishers incorporates
the publishing programmes of
D. Reidel, Martinus Nijhoff, Dr W. Junk and MTP Press.

Sold and distributed in the U.S.A. and Canada
by Kluwer Academic Publishers,
101 Philip Drive, Norwell, MA 02061, U.S.A.

In all other countries, sold and distributed
by Kluwer Academic Publishers Group,
P.O. Box 322, 3300 AH Dordrecht, The Netherlands.

Printed on acid-free paper

Softcover reprint of the 1st edition 1996

To our families and friends

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Preface

The discovery of quantum theory showed that non-commutative structures are a fundamental feature of the physical world. One of the points we would like to emphasize is that the same is true in mathematics: non-commutative structures are natural and essential in mathematics. In fact, they explain many properties of mathematical objects — at a profound level.

In this work we answer some basic questions about Lie algebras and Lie groups.

1) Given a Lie algebra, for example, in matrix terms, or by prescribed commutation relations, how can one get a realization that gives some idea of what it 'looks like.' We present a concrete theory of representations with an emphasis on techniques suitable for (efficient) symbolic computing.

2) How do classical mathematical constructs interact with Lie structures? We take stochastic processes as an example. One can think of a Lie group as a non-commutative 'black box' and map functions, such as processes or trajectories of a dynamical system via the Lie algebra, and see what comes out.

Although some familiarity with groups is prerequisite, the basic methods used to find representations do not involve advanced knowledge. In fact, this is one of our main points. This book provides the reader techniques with which doing calculus on non-commutative structures can become as basic a tool as calculus on Euclidean space is presently.

The first author would like to express appreciation to Bruno Gruber for encouragement and support in many ways. He acknowledges the participants of the Carbondale Seminar over the past few years, with special thanks to J. Kocik and M. Giering. There is much appreciation to the Université de Nancy I: especially, to the Mathematics Seminar and to the computer science department, INRIA/Lorraine, for their support of this project over several years. Finally, the first author would like to acknowledge the hospitality and fruitful discussions during several visits in Toulouse, especially G. Letac and M. Casalis.

The second author thanks M. C. Haton for accepting to chair the department during the writing of this series. He is grateful to the Mathematics Department at SIU-C for kind hospitality during several visits.

We thank Randy Hughes for his TeX expertise that was so helpful for producing this series (including the Young tableaux for volume 2). Finally, we gratefully acknowledge the support of NATO, who provided travel and per diem support for several years.