

# Unsteady Computational Fluid Dynamics in Aeronautics

# FLUID MECHANICS AND ITS APPLICATIONS

Volume 104

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P.G. Tucker

# Unsteady Computational Fluid Dynamics in Aeronautics

 Springer

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*To my Family*

# Preface

In 2001 I published ‘Computation of Unsteady Internal Flows’. This text was largely based around incompressible flow solver methods and hence typically lower speed flows. The key premise behind the original text was that, in some sense, most engineering flows are intrinsically unsteady (even if just due to turbulence). However, because of computational expense, this aspect is often ignored. Of course computing power continues to rise. The use of Graphical Processor Units for number processing is showing promise with rival technologies beginning to emerge.

Detached Eddy Simulation and related eddy resolving methods have added impetus to the use of unsteady Computational Fluid Dynamics. Simulations that potentially rival tremendously expensive rig/wind tunnel tests are now appearing. A notable shoot from this emerging era is work around 2007 at the US Airforce Laboratory, who performed DES for a F/A-18 fighter configuration. Tail buffet was explored and successful comparison made with real flight data (in terms of spectral shape of surface pressure data). This situation was not unforeseen. Around 1975, Chapman, Director of Aeronautics at NASA, proposed, using well founded scientific arguments,<sup>1</sup> that when computers reached  $10^{14}$  flops, eddy resolving simulations that could rival aerodynamic tests would emerge. Modern high performance computing provision now exceeds Chapman’s expectations, reaching Peta scale and beyond. Hence, now the ability to directly predict turbulence, for complex engineering systems, without recourse to accuracy reducing assumptions becomes ever closer—even if advances in solver technology have not been as extensive as perhaps expected by Chapman. The current text focuses on aerospace. Hence, unlike the former, it also includes discussion of compressible flow technology.

With the projected demand for air transport set to double the world aircraft fleet by 2020 it is becoming urgent to take steps to reduce environmental impact with respect to noise and other emissions. Hence, the current text, hopefully, will contribute, in some sense, to the quest to use computers to improve aircraft

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<sup>1</sup>Note, Chapman’s outer boundary layer scalings are optimistic but this aspect is less critical than the inner scalings.

and thus impact on this pressing environmental need. To make major technological breakthroughs, ultimately, extremely close airframe and engine integration will be needed. This gives the requirement for coupled engine-airframe simulations. Also, increasingly multi-physics simulations will be required. Such endeavors do not marry well with the obvious accuracy benefits provided by making turbulent eddy-resolving simulations. Hence, this text attempts to explore these tensions.

In preparing the text, great effort has been made to remove errors of a typographical nature. Apologies for the errors that are doubtless found.

I would like to express my gratitude to past Researchers who have helped run many of the simulations contained in this text. Especial thanks are due to my longest serving team members—Drs. R. Jefferson-Loveday and J. Tyacke.

The original text was prepared in WORD. Then Vadlamani Nagabhushana Rao lead an intrepid team who kindly converted the text to L<sup>A</sup>T<sub>E</sub>X, properly linking references figures and equations to the text. I am very grateful to the L<sup>A</sup>T<sub>E</sub>X team: Ahmed Al-Shabab; Zaib Ali, Jiahuan Cui, Mahak Mahak; James Page; Vadlamani Nagabhushana Rao, Robert Watson and Xiaoyu Yang.

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January 2013

Paul G. Tucker

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# Nomenclature<sup>1</sup>

## Lowercase Roman

$a_{ij}$	Anisotropy tensor
$c$	Chord, speed of sound, wave velocity, turbulence modelling constant or aspect ratios
$c'$	Pseudo acoustic speed
$c_p$	Specific heat capacity at constant pressure
$c_v$	Specific heat capacity at constant volume
$d$	Normal wall distance
$d_{ij}^{n-1}, \tilde{d}_{ij}^n$	Edge lengths in spring analogy before and after movement, respectively
$\tilde{d}$	Turbulence length scale in DES
$\tilde{d}_P$	Wall distance from Poisson equation
$f$	Frequency
$f_a$	Frequency of 1st harmonic of feedback loop (see Chap. 4)
$f_{sw}$	Body force wake activation function
$f_w, f_{v1}, f_{v2}, f_d$	Functions in the Spalart-Allmaras turbulence model
$f(\tilde{d})$	Function in Hamilton-Jacobi equation
$\tilde{f}_p, \tilde{f}_b, \tilde{f}_{KH}, \tilde{f}_{BL}$	Blade wake/disturbance, shedding, Kelvin-Helmholtz and boundary layer dimensionless frequencies
$g(d)$	Function in Hamilton-Jacobi equation
$h$	Blade thickness or heat transfer coefficient
$i, j, k$	Array or grid point location identifiers
$k$	Thermal conductivity, turbulent kinetic energy, temporal weighting function component or variable to ensure that the acoustic wave speed is similar to the particle speed

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<sup>1</sup>The nomenclature is set out as follows—First lowercase Roman letters are given followed by uppercase. Then Greek lowercase followed by uppercase symbols are given. Then superscripts and subscripts are set out. Over bars are then listed followed by special symbols and operators. Finally, the abbreviations used in the text are summarized.

$k_f = \omega c / 2U_\infty$	Reduced frequency of aerofoil pitching
$k_{ij}$	Coefficient in spring analogy
$k_p, k_I, k_D$	Free parameters in proportional integral controller
$l$	Turbulence or correlation length scale or ratio of edge lengths
$l_\mu, l_\epsilon$	Turbulence model length scales
$\dot{m}$	Mass flow rate
$n$	Number of resolved Fourier coefficients, direction cosine or number of iterations
$n_{cpl}$	Number of global solution cycles
$n_p$	Pressure switch in body force model
$p$	Static pressure, number of coefficients used in scheme or number of stages
$q$	Heat flux or solution variable
$r, \theta, z$	Cylindrical polar spatial coordinates
$r_d$	Shielding function in delayed DES
$r_e, r_n$	Exact and numerical wave numbers
$rms_\phi$	Normalised root mean square change
$s$	Transition function, streamwise distance from trailing edge, surface or surface area
$t$	Time
$t_{fp}$	Time for a fluid particle to pass through a blade passage
$t_i$	Turbulent time scale
$t_l$	Time period needed to gain a satisfactory average
$t_p$	Periodic time scale
$t_r = t -  x - y /c$	Retarded time
$t_t$	Transient time scale
$u, v, w$	Instantaneous $x, y, z$ , velocity components
$w$	Wave number, velocity component or wake half width
$w_c$	Cut-off wave number
$w'$	Numerical wave number
$x, y, z$	Spatial coordinates
<i>Uppercase Roman</i>	
$A$	Global representation of spatial discretization, central nodal coefficient, amplitude or Roe matrix
$\hat{A}$	Root mean square of quantity in correlation coefficient for zero space and time separations
$A_\mu, A_\epsilon$	Turbulence model constants
$A_\omega$	Average cross sectional area normal to vorticity vector
$B$	Amplitude
$C$	Courant number ( $u \Delta t / \Delta x$ ), constant or amplitude
$C_s$	Smagorinsky constant
$C_l, C_\mu, C_w$	Turbulence model constants
$C_t$	Safety factor
$C_{\Delta t}$	Time domain objective
$C_{ij}$	Clark terms



$C_{pt}$	Total pressure loss coefficient
$C_D$	Drag coefficient
$C_f$	Skin friction coefficient
$C_M$	Moment coefficient
$C_p$	Surface pressure coefficient
$D$	Time step to diffusion time scale ratio or diameter scale
$E$	Solution error, flux term or energy in turbulence energy spectrum
$F_p, F_n$	Forces parallel and normal to blade passages, respectively
$[F_S]$	Force matrix
$F$	Flux term
$F_{SST}$	Delayed DES function in Menter SST framework
$G$	Flux, grid distortion term or filter kernel/operator
$H$	Representation of step height or closing disc extent
$H(x, \bar{\Delta}, \tau)$	Correlation between velocity and temperature located at $x$
$I$	Second moment of area tensor
$I_o$	Principal axis system
$Id$	Identity matrix
$J$	Jacobian or flux component
$K_n, K_{vd}, K_w, K_l, K_p$	Wake body force model calibration constants
$K_0$	Constant in Kolmogorov energy spectrum
$[K_s]$	Stiffness matrix
$[K_f]$	Fluid system matrix
$L$	Length scale, integration range, linear turbulent stress component or wave operator
$L_{vK}$	von Karman length scale
$\tilde{L}$	Length scale in Hamilton-Jacobi equation
$L_{ij}$	Leonard terms
$m$	Order of scheme
$M$	Mach number or capacitance or mass operator/matrix
$N$	Number of mesh points, realizations or blades
$Nb$	Number of blades
$N_{CD}$	Order of central difference scheme
$N_f$	Number of Fourier modes
$\overline{NL}$	Non-linear turbulent stress component
$N_{NB}$	Marker zone variable in zonal hybrid RANS-LES
$N_{P\phi}$	Number of mesh points for each solution variable
$N_{UP}$	Order of upwind scheme
$Nu = qL/k\Delta T$	Nusselt number
$N_\phi$	Number of solution variables
$P$	Number of processors
$P_k, P_{shear}$	Production of turbulence energy
$Pr = \mu c_p/k$	Prandtl number
$\underline{Q}$	Jacobian or under-relaxation function
$R$	Gas constant, radius scale or residual

$Re$	Reynolds number
$R(x, \overline{\Delta}, \tau)$	Correlation coefficient, for velocity, located at $x$
$[R]$	Coupling matrix
$S$	Source or strain term
$S_{ij}$	Mean strain rate tensor
$St$	Strouhal number
$T$	Temperature
$\mathbf{T}$	Transformation matrix
$Te$	Execution time
$Ti$	Turbulence intensity
$T_{ij}$	Lighthill stress tensor
$TV$	Total variation
$U, U_\theta$	Blade and fluid tangential velocities, respectively
$U, V, W$	Instantaneous contravariant velocity components
$[U]$	Displacement matrix
$U_c$	Bulk or convection velocity
$U_o$	Bulk velocity
$u_\tau$	Friction velocity
$U_\infty$	Free stream velocity
$V$	Relative velocity in a blade passage, general velocity scale or volume
$Vol$	Cell volume
$Wf$	Weighting function
$X, Y, Z$	Dimensionless spatial coordinates
<i>Lowercase Greek</i>	
$\alpha$	Dimensionless weighting parameter, phase angle, latency parameter in LNS model, design variable, fraction of rotor speed at which stall modes rotate in compressor (Chap. 4), blade metal angle (Chap. 4), angle of attack
$\beta$	Optimization range, weighting control or compressibility parameter
$\gamma = c_p/c_v$	Ratio of specific heats, weighting parameter in compact scheme or intermittency
$\delta$	Boundary layer thickness, step function or small number/perturbation
$\epsilon$	Turbulence dissipation rate, small number, scaling parameter in HJ equation, (specified) error tolerance/level or flux limiter
$\epsilon_0$	Positive relaxation parameter
$\epsilon_1$	Numerical smoothing parameter
$\eta$	Parameter that defines time levels in discretized equations, transformed spatial variable or adiabatic film cooling effectiveness
$\theta$	Momentum thickness or angle
$\kappa$	von Karman constant

$\lambda$	Temporal discretization control parameter, Eigen values, viscosity coefficient ( $-2\mu/3$ ), wave speed (in LES filter definition) or length, or inverse velocity scale
$\mu$	Dynamic viscosity
$\mu_t$	Turbulent viscosity
$\nu$	Kinematic viscosity
$\nu_t$	Turbulent kinematic viscosity
$\xi, \eta, \zeta$	General, transformed coordinates
$\rho$	Fluid density
$\sigma$	Diffusion Prandtl number or turbulence fluctuation scale
$\tau$	Transformed temporal coordinate, shear stress, turbulence time scale or time shift
$\tau_s$	Correlation time scale
$\phi$	General variable
$\psi$	Difference between current value and a predefined dispersion level, ratio of mean flow and turbulence time scales
$\omega$	Frequency (turbulence) or vorticity

*Uppercase Greek*

$\Gamma$	Diffusion coefficient or Jacobian matrix
$\Delta$	Filter width
$\Delta t$	Time-step length
$\overline{\Delta}$	Space shift in turbulence correlation coefficients (Chap. 6)
$\Lambda$	Adjoint variable
$\Phi$	General variable or shock switch parameter
$\Omega$	Angular velocity

*Superscripts*

<i>cen</i>	Pertaining to a central difference
<i>dis</i>	Associated with dissipation
<i>H</i>	High order component
<i>inv</i>	Pertaining to inviscid components
<i>l</i>	Index
<i>L</i>	Low order component or lower boundary (Chap. 4)
<i>n</i>	Time level
<i>new</i>	Latest value
<i>old</i>	Previous value
<i>U</i>	Upper boundary (Chap. 4)
<i>vis</i>	Pertaining to viscous components
$\Delta T$	Variable computed with a coarse time step
<i>'</i>	Perturbation or first derivate of variable
<i>''</i>	Second derivative of variable or coherent, low frequency, unsteadiness scale amplitude
$+, *$	Dimensionless distance in wall units or dimensionless variable

*Subscripts*

<i>a</i>	Pertaining to actual value
<i>ave</i>	Average value
<i>c</i>	Pertaining to chord or centroid
<i>cl</i>	Centerline value
<i>conv</i>	Convective flux component
<i>ctr</i>	Pertaining to central difference scheme
<i>DB</i>	Pertaining to database
<i>DES</i>	Pertaining to the DES model
<i>exp</i>	Pertaining to experimental value
<i>ERROR</i>	Pertaining to error
<i>f</i>	Unsteadiness associated with vacillation
<i>fp</i>	Relating to a particular moving fluid particle
<i>g</i>	Pertaining to grid
<i>HJ</i>	Pertaining to HJ equation
<i>i, j, k</i>	Array subscripts pertaining to the axial, radial and tangential directions, respectively
<i>int</i>	Interface value in hybrid RANS-LES
<i>IGV</i>	Pertaining to IGV
<i>k</i>	Pertaining to turbulence kinetic energy or midpoint of a face
<i>k-<math>\omega</math></i>	Pertaining to <i>k-<math>\omega</math></i> turbulence model
<i>l</i>	Index
<i>L</i>	Pertaining to information on the left or large eddy scales
<i>LES</i>	Pertaining to LES model
<i>max</i>	Maximum value
<i>min</i>	Minimum value
<i>num</i>	Pertaining to numerical value
<i>NB</i>	Neighboring values
<i>o</i>	Reference value or pertaining to centre of gravity
<i>P</i>	Process linked to a particular variable
<i>R</i>	Pertaining to information on the right or rotor (Chap. 4)
<i>RANS</i>	Pertaining to RANS model
<i>smth</i>	Smoothing component
<i>s</i>	Pertaining to solid
<i>S</i>	Pertaining to small turbulent scales, stator (Chap. 4)
<i>SCL</i>	Pertaining to quantity obeying the Space Conservation Law
<i>SGS</i>	Pertaining to the subgrid scale
<i>target</i>	Target value
<i>u, v, w</i>	Pertaining to listed velocity components
<i>x, y, z</i>	Pertaining to the <i>x</i> , <i>y</i> and <i>z</i> directions, respectively
<i>z, r, <math>\theta</math></i>	Pertaining to the axial, radial and tangential directions respectively
$\epsilon$	Pertaining to $\epsilon$
$\theta$	Pertaining to temperature fluctuations
$\phi$	Pertaining to the variable $\phi$
$\Delta t$	Variable represented on a finer temporal ‘grid’ of $\Delta t$

*Overbars*

$\sim$	Variable expressed in the frequency domain or that is dimensionless
$\bar{\phantom{x}}$	Averaged or filtered value
$\rightarrow$	Coarser filtered value
$\hat{\phantom{x}}$	Dimensionless variable

*Special Symbols/Operators*

int	Round up value to the nearest integer
$\Im$	Imaginary part
max	Operator to take maximum value
$N(a, b)$	Normally distributed random number operator with mean $a$ and standard deviation $b$
$NS(\phi), NS^s(\phi)$	Navier-Stokes and steady Navier-Stokes operator
$\Re$	Real part
$URANS(\phi)$	Unsteady RANS operator
$\delta(\varphi)$	Dirac delta function
$\delta_{ij}$	Kronecker delta ( $\delta_{ij} = 1$ if $i = j$ and $\delta_{ij} = 0$ if $i \neq j$ )
$\varepsilon_{ijk}$	Alternating symbol
$\langle \phantom{x} \rangle$	Phase averaged variable

*Abbreviations*

ADI	Alternating Direct Implicit
ALE	Arbitrary Eulerian-Lagrangian
APE	Acoustic Perturbation Equations
AUSM	Advection Upstream Splitting Method
BEM	Boundary Element Method
BD	Backwards Difference
Bi-CGSTAB	BiConjugate Gradient STABILized method
CAA	Computational AeroAcoustics
CABARET	Compact Accurately Boundary-Adjusting high-REsolution Technique
CD	Central difference scheme
CEN2	Second order central difference scheme
CFD	Computational Fluid Dynamics
CN	Crank-Nicolson scheme
COM6	Compact 6th Order Scheme
CPR	Correction Procedure via Reconstruction
DES	Detached Eddy Simulation
DG	Discontinuous Galerkin
DNS	Direct Numerical Simulation
DRP	Dispersion Relation Preserving
DSM	Deterministic Stress Model
EE	Euler Equation
ENO	Essentially Non-Oscillator
FCT	Flux Corrected Transport
FE	Finite Element

FRAM	Filtering Remedy And Methodology
FWH	Ffowcs-Williams and Hawkings
GMRES	Generalized Minimum RESidual
HJ	Hamilton-Jacobi
HPT	High-Pressure Turbine
HPTR	HPT-Rotor
IBA	Interpolation Based Algorithm
ILES	Implicit Large Eddy Simulation
IPT	Intermediate Pressure Turbine
IPTS	IPT-Stator
LDA	Laser Doppler Anemometry
LDDRK	Low Dissipation and Dispersion Runge-Kutta
LEE	Linearized Euler Equation
LES	Large Eddy Simulation
LNS	Limited Numerical Scales
LNSE	Linearized Navier-Stokes Equations
LPT	Low-Pressure Turbine
MDICE	MultiDisciplinary Computing Environment
MILES	Monotone Integrated Large Eddy Simulation
MUSCL	Monotone Upstream-centred; Schemes for Conservation Laws
MST	Mean Source Terms
NLAS	Non-linear Acoustics Solver
NLDE	Nonlinear Disturbance Equation
NLES	Numerical Large Eddy Simulation
NSS	Nearest Surface Search
NURBS	Non-Uniform Rational B-Splines
OASPL	Overall Sound Pressure Level
ODE	Ordinary Differential Equation
OGV	Outlet Guide Vane
PANS	Partially Averaged Navier-Stokes
PI	Proportional Integral
PIV	Particle Image Velocimetry
PPW	Points Per Wave
PSD	Power Spectral Density
RANS	Reynolds Averaged Navier-Stokes
RK	Runge-Kutta scheme
RO	Reduced Order
ROM	Reduced Order Model
RPM	Random Particle Mesh
SAS	Scale Adaptive-Simulation
SCL	Space Conservation Law
SD	Spectral Difference
SHASTA	SHarp And Smooth Transport Algorithm
SST	Shear Stress Transport

SV	Spectral Volume
T-S	Tollmien-Schlichting
TVD	Total Variation Diminishing
UP	Upwind Scheme
URANS	Unsteady Reynolds Averaged Navier-Stokes
VLES	Very Large Eddy Simulation
WALE	Wall Adapting Local Eddy-viscosity
WENO	Weighted Essentially Non-Oscillatory