

Solid Mechanics and Its Applications

Volume 202

Series Editor

G. M. L. Gladwell

Department of Civil Engineering, University of Waterloo, Waterloo, Canada

For further volumes:

<http://www.springer.com/series/6557>

Aims and Scope of the Series

The fundamental questions arising in mechanics are: *Why? How? and How much?* The aim of this series is to provide lucid accounts written by authoritative researchers giving vision and insight in answering these questions on the subject of mechanics as it relates to solids.

The scope of the series covers the entire spectrum of solid mechanics. Thus it includes the foundation of mechanics; variational formulations; computational mechanics; statics, kinematics and dynamics of rigid and elastic bodies; vibrations of solids and structures; dynamical systems and chaos; the theories of elasticity, plasticity and viscoelasticity; composite materials; rods, beams, shells and membranes; structural control and stability; soils, rocks and geomechanics; fracture; tribology; experimental mechanics; biomechanics and machine design.

The median level of presentation is the first year graduate student. Some texts are monographs defining the current state of the field; others are accessible to final year undergraduates; but essentially the emphasis is on readability and clarity.

Seyed Habibollah Hashemi Kachapi
Davood Domairry Ganji

Dynamics and Vibrations

Progress in Nonlinear Analysis

 Springer

Seyed Habibollah Hashemi Kachapi
Department of Mechanical Engineering
Babol University of Technology
Babol
Iran

Davood Domairry Ganji
Department of Mechanical Engineering
Babol University of Technology
Babol
Iran

ISSN 0925-0042

ISBN 978-94-007-6774-4

ISBN 978-94-007-6775-1 (eBook)

DOI 10.1007/978-94-007-6775-1

Springer Dordrecht Heidelberg New York London

Library of Congress Control Number: 2013944670

© Springer Science+Business Media B.V. 2014

This work is subject to copyright. All rights are reserved by the Publisher, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilms or in any other physical way, and transmission or information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed. Exempted from this legal reservation are brief excerpts in connection with reviews or scholarly analysis or material supplied specifically for the purpose of being entered and executed on a computer system, for exclusive use by the purchaser of the work. Duplication of this publication or parts thereof is permitted only under the provisions of the Copyright Law of the Publisher's location, in its current version, and permission for use must always be obtained from Springer. Permissions for use may be obtained through RightsLink at the Copyright Clearance Center. Violations are liable to prosecution under the respective Copyright Law. The use of general descriptive names, registered names, trademarks, service marks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

While the advice and information in this book are believed to be true and accurate at the date of publication, neither the authors nor the editors nor the publisher can accept any legal responsibility for any errors or omissions that may be made. The publisher makes no warranty, express or implied, with respect to the material contained herein.

Printed on acid-free paper

Springer is part of Springer Science+Business Media (www.springer.com)

Preface

Introduction

Dynamical and vibratory systems are basically applications of mathematics and science to the solution of real-world problems. In the majority of real-life and applied phenomena in engineering sciences, as well as in a multiplicity of other sciences, solutions of specifically defined problems are the ultimate goal. In order to apply engineering or any other science, it is necessary to fully understand dynamical and vibratory systems and how to solve cases of either linear or nonlinear equations using analytical and numerical methods. It is of particular importance to study nonlinearity in dynamics and vibration, because almost all applied processes act nonlinearly. In addition, nonlinear analysis of complex systems is one of the most important and complicated tasks, especially in engineering and applied science problems.

There are only a handful of books that focus on nonlinear dynamics and vibrations analysis. Some of these books are written at a fundamental level that may not meet ambitious engineering program requirements. Others are specialized in certain fields of oscillatory systems, including modeling and simulation. In this book, we attempt to strike a balance between theory and practice, fundamentals and advanced subjects, and generality and specialization.

None of the books in this area have completely studied and analyzed nonlinear equations in dynamical and vibratory systems using the latest analytical and numerical methods, which, if included, would allow the user to solve problems without needing to study many different references. Therefore, in this book, we have chosen to use the latest analytic and numerical laboratory methods, referring to a bibliography of more than 300 books, papers, and research reports, many of them written by the authors of this book, and to consider almost all possible processes and physical configurations, thereby exploring new theories that have been proposed to solve real-life problems in engineering and applied sciences. In this way, the users (bachelor's, master's, and Ph.D. students, university teachers, and even workers in research centers in different fields of mechanical, civil, aerospace, electrical, chemical, applied mathematics, physics, etc.) can approach such systems with confidence. In the different chapters of the book, not only are

linear and nonlinear problems, especially those in an oscillatory form, broadly discussed, but also applied examples are solved in a practical manner by the proposed methodology.

An abundant number of examples and homework problems are provided.

The users of this collection can achieve very strong capabilities in the area, especially of nonlinear phenomena in dynamically and vibratory systems, such as the following:

- A complete understanding of the nonlinearity sources and formulation of dynamical motion equations in different systems using the most general methods (e.g., principle of virtual work, D'Alembert's principle, Newton and Lagrange methods, etc.).
- A complete understanding of the fundamentals of oscillatory systems and their governing nonlinear equations; also analytical and numerical methods in solving applied problems, especially those with nonlinearities.
- A complete study of mathematical problems in engineering, analytic, and numeric methods (e.g., perturbation methods, the homotopy perturbation method, variational methods, energy methods, limit cycles, the parameterized perturbation method, the singular perturbation method, Adomian's decomposition method, the differential transformation method and its modification, He's parameter expansion method, He's amplitude–frequency formulation, the harmonic balance method, the coupled method of homotopy perturbation, the variational method, Floquet theory, etc.).
- Complete familiarity with specialized processes and applications in different areas of the field, studying them, eliminating complexities and controlling them, and also applying them in real-life engineering cases.
- A complete analysis of important engineering systems (e.g., NDOF systems, discs, springs, beams, normal modes, multibody phenomena, shafts, sliders, the human body, nonlinear oscillators in automobile design, rotating rigid frames, flexible beams, rotating rigid hubs, elastic cantilever beams, the human eardrum, etc.).
- A complete analysis of important equations in the field and their generalizations in real-life applications with practical examples (Duffing's oscillation, Van der Pol's oscillation, Mathieu's oscillation, Hamiltonian oscillation, Hill's oscillation, resonances, viscoelasticity, damping, fraction order, cubic nonlinearity, coupled systems, wave equations, etc.).
- The ability to encounter, model, and interpret an engineering process or system and to solve the related complexities engendered by the vibrations property in linear and nonlinear cases.

Audience

This book is a comprehensive and complete text on dynamical and vibratory motions and analytical and numerical methods in applied problems. It is self-contained, and the subject matter is presented in an organized and systematic manner. This book is quite appropriate for several groups of people, including the following:

- Senior undergraduate and graduate students taking courses in the mentioned fields.
- Professionals, for whom the book can be adapted for a short course on the subject matter.
- Design and research engineers, who will be able to draw upon the book in selecting and developing mathematical models for analytical and design purposes in applied conditions.
- Practicing engineers and managers who want to learn about the basic principles and concepts involved in the solving of problems using analytical and numerical methods such as dynamics, vibrations, and systems analysis and how they can be applied at their own workplaces.
- Generally, users who are bachelor's, master's, and Ph.D. students, university teachers, and even researchers at centers in different fields of mechanical, civil, aerospace engineering, applied physics, mathematics, and so forth.

Because the book is aimed at a wide audience, the level of mathematics is kept intentionally low. All the principles presented in the book are illustrated by numerous worked examples. The book draws a balance between theory and practice.

Acknowledgments

We are grateful to all those who have had a direct impact on this work. Many people working in the general areas of nonlinear phenomena, vibrations, oscillations, dynamics, mathematical, and physical problems, and analytical and numerical methods have influenced the format of this book.

The authors are very thankful to Babol Noshirvani University of Technology, Iran, and the National Elite Foundation of Iran (Bonyad Melli Nokhbeghan), Mazandaran Province, Sari, Iran, especially the nonlinear dynamics teams in the Mechanical Engineering Department, and all professors and students of all Iranian universities who helped them develop research skill, editing the electronic text, giving them useful consultation and precious guidance, and providing references for the authors, especially examples of applications that were used in different chapters.

The authors are thankful to the Mechanical Engineering Department of Fairfield University, USA, especially deceased Prof. Rao V. Dukkipati, and Prof. Ji-Huan He (Donghua University, Modern Textile Institute, Shanghai, China), and are especially thankful to the Department of Mathematics of Saint Xavier University, Chicago, USA, and to Professor Abdul-Majid Wazwaz for helping them develop their research skills, editing the electronic text and giving useful consultations, and also for his precious guidance and for the references and lectures provided to the authors for use in different chapters.

We are indebted to many colleagues and to numerous authors who have made contributions to the literature in this field. In addition, I am greatly indebted to all the authors of the articles listed in the bibliography of this book, such as Professors Ali Nayfeh, Shaher Momani, Liao, G. Adomian, J. Srinivas, Richard H. Rand, Alexander Fidlin, and other professors for their kind and faithful support and references that were directly used in the writing of this book. I would also like to thank the reviewers for their efforts and for their comments and suggestions, which have well served to compile the best possible book for the intended and targeted audience.

A special acknowledgment is expressed to the chief editor of the publisher for his excellent revision of the English language of the book and for editing the electronic text.

Finally, we would very much like to acknowledge the devotion, encouragement, patience, and support provided by our family members.

I would appreciate being informed of errors or receiving other comments about the book. Please write to the authors at the Babol Noshirvani University of Technology address or send e-mail to: sha.hashemi.kachapi@gmail.com (Seyed H. Hashemi Kachapi) and ddg_davood@yahoo.com (D. D. Ganji)

We sincerely hope that the final outcome of this book will help students, researchers, and other users in developing an appreciation for the topic of analysis of nonlinear dynamical systems and nonlinear vibration analysis using analytical and numerical methods.

Contents

1	Introduction to Nonlinear Vibrations and Dynamics	1
1.1	Usual Sources of Nonlinearity in Mechanical and Other Engineering	1
1.1.1	Introduction	1
1.1.2	Geometrical Nonlinearities	1
1.1.3	Physical Nonlinearities	2
1.1.4	Structural or Designed Nonlinearities	3
1.1.5	Constraints	4
1.1.6	Nonlinearity of Friction	5
1.2	Formulation of Equations	7
1.2.1	Introduction	7
1.2.2	Principle of Virtual Work	7
1.2.3	d'Alembert's Principle	11
1.2.4	Lagrange's Equations of Motion	12
1.2.5	Newton's Method	16
1.3	Applied Examples	18
	References	48
2	Perturbation and Variational Methods	49
2.1	Introduction	49
2.2	The Basic Ideas of Perturbation Analysis	50
2.2.1	Variation of Free Constants and Systems in Standard Form	51
2.2.2	Standard Averaging as an Almost Identical Transformation	52
2.2.3	Method of Multiple Scales	55
2.2.4	Direct Separation of Motions	57
2.2.5	Relationship Between These Methods	58
2.2.6	Application	59
2.2.7	Introduction	61
2.2.8	The Method of Multiple Scales	61

- 2.3 Parameterized Perturbation Method 69
 - 2.3.1 Introduction 69
 - 2.3.2 Application 70
- 2.4 Singular Perturbation Method 71
 - 2.4.1 Introduction 71
 - 2.4.2 Application 72
- 2.5 Homotopy Perturbation Method and Its Modifications 74
 - 2.5.1 A Brief Introduction to the Homotopy
Perturbation Method 74
 - 2.5.2 Application 78
- 2.6 Variational Iteration Method 90
 - 2.6.1 Introduction 90
 - 2.6.2 Application 92
- 2.7 He’s Variational Approach 96
 - 2.7.1 Basic Idea 96
 - 2.7.2 Application 98
- 2.8 Couple Variational Method 105
 - 2.8.1 Introduction 105
 - 2.8.2 Application 105
- 2.9 Energy Balance Method 108
 - 2.9.1 Introduction 108
 - 2.9.2 Application 110
- 2.10 Coupled Method of Homotopy Perturbation
and Variational Method 115
 - 2.10.1 Introduction 115
 - 2.10.2 Application 116
- References 129

- 3 Considerable Analytical Methods 133**
 - 3.1 Harmonic Balance Method 133
 - 3.1.1 Introduction 133
 - 3.1.2 Governing Equation of Motion and Formulation 134
 - 3.1.3 First-Order Analytical Approximation 136
 - 3.1.4 Second-Order Analytical Approximation 137
 - 3.1.5 Third-Order Analytical Approximation 138
 - 3.1.6 Approximate Results and Discussion 139
 - 3.2 He’s Parameter Expansion Method 141
 - 3.2.1 Introduction 141
 - 3.2.2 Modified Lindstedt–Poincaré Method 142
 - 3.2.3 Bookkeeping Parameter Method 142
 - 3.2.4 Application 142
 - 3.2.5 Governing Equation 144
 - 3.2.6 HPEM for Solving Problem 145

3.3	Differential Transformation Method	146
3.3.1	Introduction	146
3.3.2	Differential Transformation Method	147
3.3.3	Padé Approximations.	149
3.3.4	Application	150
3.4	Adomian’s Decomposition Method	154
3.4.1	Basic Idea of Adomian’s Decomposition Method	154
3.4.2	Application	156
3.5	He’s Amplitude–Frequency Formulation.	161
3.5.1	Introduction	161
3.5.2	Applications	162
3.5.3	Problems	167
	References	181
4	Introduction of Considerable Oscillatory Systems	185
4.1	Duffing’s Oscillation Systems	185
4.1.1	Introduction to Duffing’s Oscillation	185
4.1.2	The Forced Duffing Oscillator	192
4.1.3	Universalization and Superposition in Duffing’s Oscillator.	198
4.2	The Van der Pol Oscillator Systems	203
4.2.1	The Unforced Van der Pol Oscillator	203
4.2.2	The Forced Van der Pol Oscillator	208
4.2.3	Two Coupled Limit Cycle Oscillators	214
4.3	Mathieu’s Equation	220
4.3.1	Introduction	220
4.3.2	Effect of Damping	229
4.3.3	Effect of Nonlinearity	230
4.4	Ince’s Equation	233
4.4.1	Introduction	233
4.4.2	Coexistence	234
4.4.3	Ince’s Equation.	236
4.4.4	Designing a System with a Finite Number of Tongues.	239
4.4.5	Application	240
	References	247
5	Applied Problems in Dynamical Systems.	249
5.1	Problem 5.1. Displacement of the Human Eardrum	249
5.1.1	Introduction	249
5.1.2	Variational Iteration Method.	249
5.1.3	Perturbation Method	250

5.1.4	Homotopy Perturbation Method	252
5.1.5	Numerical Solution	253
5.2	Problem 5.2. Slides Motion Along a Bending Wire	254
5.2.1	Introduction	254
5.2.2	Energy Balance Method.	255
5.2.3	Variational Iteration Method.	256
5.2.4	Parameter Lindstedt–Poincaré Method.	257
5.3	Problem 5.3. Movement of a Mass Along a Circle	260
5.3.1	Introduction	260
5.3.2	Energy Balance Method.	261
5.3.3	Variational Iteration Method.	262
5.3.4	Parameter Lindstedt–Poincaré Method.	263
5.4	Problem 5.4. Rolling a Cylinder on a Cylindrical Surface	265
5.4.1	Introduction	265
5.4.2	Energy Balance Method Results	266
5.4.3	Variational Iteration Method Results	267
5.4.4	Parameter Lindstedt–Poincaré Method Results	267
5.5	Problem 5.5. Movement of Rigid Rods on a Circular Surface.	268
5.5.1	Introduction	268
5.5.2	Energy Balance Method.	269
5.5.3	Variational Iteration Method.	270
5.5.4	Parametrized Perturbation Method.	272
5.6	Problem 5.6. Application of Two Degrees of Freedom Viscously Damped.	275
5.6.1	Introduction	275
5.6.2	Application of the Homotopy Perturbation Method.	276
5.7	Problem 5.7. Application of Viscous Damping with a Nonlinear Spring	282
5.7.1	Introduction	282
5.7.2	Application of Homotopy Perturbation Method.	283
5.7.3	Underdamped System $\left(\zeta^2 < 1 \text{ or } \frac{c}{2m} < \sqrt{\frac{k}{m}}\right)$	285
5.7.4	Overdamped System $\left(\zeta^2 > 1 \text{ or } \frac{c}{2m} > \sqrt{\frac{k}{m}}\right)$	288
5.7.5	Critically Damped System $\left(\zeta^2 = 1 \text{ or } \frac{c}{2m} = \sqrt{\frac{k}{m}}\right)$	292
5.7.6	Discussion and Conclusion.	294
5.8	Problem 5.8. Application of Cubic Nonlinearity	297
5.8.1	Introduction	297
5.8.2	First Assumption.	299
5.8.3	Second Assumption.	303
5.9	Problem 5.9. Van der Pol Oscillator	305
5.9.1	Introduction	305
5.9.2	The Application of PM in the Van der Pol Oscillator	306
5.9.3	Homotopy Perturbation Method	307

- 5.9.4 Application of VIM in the Van der Pol Oscillator. 308
- 5.9.5 Application of ADM in the Van der Pol Oscillator 309
- 5.10 Problem 5.10. Application of a Slender, Elastic Cantilever Beam 313
 - 5.10.1 Introduction 313
 - 5.10.2 Solution Using the First Case of the Homotopy Perturbation Method 315
 - 5.10.3 Solution Using Second Case of the Homotopy Perturbation Method 320
- 5.11 Problem 5.11. Dynamic Behavior of a Flexible Beam Attached to a Rotating Rigid Hub 322
 - 5.11.1 Introduction 322
 - 5.11.2 Application of the Homotopy Perturbation Method 323
 - 5.11.3 Application of the Energy Balance Method 325
 - 5.11.4 Results. 326
- 5.12 Problem 5.12. The Motion of a Ring Sliding Freely on a Rotating Wire 326
 - 5.12.1 Introduction 326
 - 5.12.2 Application of HPEM 328
- 5.13 Problem 5.13. Application of a Rotating Rigid Frame Under Force 330
 - 5.13.1 Introduction of Case 1 330
 - 5.13.2 Application of HPEM 330
 - 5.13.3 Introduction of Case 2 332
 - 5.13.4 Solution of Case 2 Using Frequency Formulation 333
- 5.14 Problem 5.14. Application of a Nonlinear Oscillator in Automobile Design 335
 - 5.14.1 Introduction 335
 - 5.14.2 Solution Using the Amplitude Frequency Formulation 336

Notation and Units

Both the SI and the US/English systems of units have been used throughout the book.