Solid Mechanics and Its Applications

Volume 198

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The median level of presentation is the first year graduate student. Some texts are monographs defining the current state of the field; others are accessible to final year undergraduates; but essentially the emphasis is on readability and clarity.

André Preumont

Twelve Lectures on Structural Dynamics

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Il n'y a que la vérité qui persuade, même sans avoir besoin de paraître avec toutes les preuves.
Elle entre si naturellement dans l'esprit, que quand on l'apprend pour la première fois, il semble qu'on ne fasse que s'en souvenir.

Fontenelle
Entretiens sur la pluralité des mondes (1686)

Preface

Nowadays, with the amazing computing capability of computers and the availability of sophisticated, user-friendly computer-aided analysis software, the main difficulty for the analyst is to interpret the results and to make sure that the analysis includes all the relevant physical phenomena. The majority of structural failures occur because physical phenomena are overlooked, or greatly underestimated, rather than as a result of computational errors (e.g., the flutter of the Takoma suspension bridge, or the recent Fukushima tsunami disaster). To build confidence in the design, the analyst first develops a crude model, a model of minimum complexity which still reflects the main physical phenomena. What minimum complexity means depends on the problem; for example, a point mass with two degrees of freedom is sufficient to explain the stable operation of a rotor at supercritical velocities, but it is impossible to account for the dependency of the natural frequencies on the rotor speed without including the gyroscopic effects; the effect of prestresses on the natural frequencies cannot be accounted for without the inclusion of nonlinear strains (Green strain tensor). Similarly, analytical results may appear unnecessary at a time where extensive parametric studies may be performed numerically very quickly, but knowledge of the parametric dependence of critical properties such as natural frequency on design parameters (dimensions, material properties,…) is invaluable in design, especially when scale effects are involved.

This book is based on the vibration course that I teach in the joint masters program ULB/VUB at the university of Brussels, and the random vibration course that I taught for a decade at the university of Liege. It has also been strongly influenced by my research work in the active control of structures, where we usually work with models of moderate size, but which accounts for all the relevant features of the system dynamics. The book is focused on modeling, with very little, if any, attention paid to the numerical methods that I consider as mature and well established, and which are well covered in the excellent books written by those more qualified than me (e.g., Geradin and Rixen).

I would like to pay a tribute to my own teachers at the university of Liege where I was initiated into structural analysis. Some 40 years later, I still have vivid memories of the sparkling lectures of Prof. Ch. Massonnet on the strength of materials and of Prof. B. Fraeijs de Veubeke on vibrations (all without notes); and later Michel Geradin advised me during my doctorate. Reading the wonderful books from S. H. Crandall; Y. C. Fung; Y. K. Lin; and L. Meirovitch, and a few others also made a lasting impression on me. Finally, I wish to thank my coworkers at ULB for their enthusiasm and raising many questions that I have attempted to answer in this book. A special thanks goes to Renaud Bastaits who helped me to produce these notes, and did artfully most of the figures.

Bruxelles, December 2012 André Preumont

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