Physics of Transitional Shear Flows

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Physics of Transitional Shear Flows

Instability and Laminar–Turbulent Transition in Incompressible Near-Wall Shear Layers

Springer

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'… it is far easier to produce numbers from a computer than understand their relevance to real flows.'

P. G. Saffman & G. R. Baker, Ann. Rev. Fluid Mech.*, 1979.*

Foreword

Hydrodynamic instability and transition to turbulence in boundary layers has been an intensive research interest during the last century and continues to be so in the 21st century. There is no dearth of research results on the topic and many contributions appear annually in journal papers and conference proceedings. As a consequence, the laminar-turbulent transition problem is a manifold branch of fluid mechanics with a variety of theoretical and experimental approaches to basic and engineering issues that are still open. Naturally, there is a need to summarize from time to time the current state in transition studies through review papers and monographs. That is the goal of this book presented by the Novosibirsk group of researchers who are well-experienced in long-term experimental modeling of transition physics in boundary layers. In fact, the Novosibirsk group, centered at the Institute of Theoretical and Applied Mechanics, has been amongst the leaders in foundational contributions to the area for decades and it is therefore appropriate and welcome that they contribute a monograph on the topic.

Starting from fundamentals of classical stability theory, the authors give an overview of the transition phenomena in subsonic, wall-bounded shear flows. At first, the consideration focuses on elementary small-amplitude velocity perturbations of laminar shear layers, i.e. instability waves, in the simplest canonical configurations of a plane channel flow and a flat-plate boundary layer. Then the linear stability problem is expanded to include the effects of pressure gradients, flow curvature, boundary-layer separation, wall compliance, etc. related to applications. Beyond the amplification of instability waves is the non-modal growth of local stationary and non-stationary shear flow perturbations which are discussed as well. Moreover, the authors proceed with the key aspect of the transition process, that is, receptivity of convectively unstable shear layers to external perturbations, summarizing main paths of the excitation of laminar flow disturbances. The remainder of the book addresses the instability phenomena found at late stages of transition. These include secondary instabilities and nonlinear features of boundary-layer perturbations that lead to the final breakdown to turbulence. Thus, the reader is provided with a step-by-step approach that covers the milestones and recent advances in the laminar-turbulent transition.

Special aspects of instability and transition are discussed through the book and are intended for research scientists, while the main target of the book is the student in the fundamentals of fluid mechanics. Computational guides, recommended exercises, and PowerPoint multimedia notes based on results of real scientific experiments supplement the monograph. These are especially helpful for the neophyte to obtain a solid foundation in hydrodynamic stability.

William S. Saric PhD, NAE
University Distinguished Professor
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Preface

Overview

The origin of turbulence in fluids is a long-standing problem that has been the focus of research for decades due to its great importance in a variety of engineering applications. Moreover, studying of onset of turbulence is part of the fundamental physical problem of turbulence description and the philosophical problem of determinism and chaos.

In the present book we try to give a panoramic view of the transition of turbulence in near-wall shear flows by binding its different aspects for incompressible flows according to our knowledge obtained during decades of researches. To be in line with this general idea, there was a need to pick out the material for inclusion from plenty of available sources. In this way, our priority was a conceptual treatment of various both classical and modern aspects of the problems under consideration.

The scope and the resulting structure of the book were chosen based on experience collected owing to one semester lecture courses for PhD students read by some of us at Novosibirsk State University, Korean Advanced Institute of Science and Technology, and Pusan National University.

Thus, the book is intended first of all for beginners in this field of science. We expect that the result of our efforts will also be of interest to researchers and high-school teachers involved in hydro-aerodynamics, stability, and laminar–turbulent transition problems.

Prerequisites

Readers of this book are expected to be familiar with the concepts of fluid mechanics of viscous incompressible flow and Prandtl boundary-layer theory. It is also desirable, but not required, for the reader to know basic ideas of linear algebra and to have some experience with matrix computations and MATLAB programming.

Organization of the Book

In Chaps. 1 and 2, we begin with basic observations and the fundamentals of the classical stability theory, which describes low-intensity shear-layer perturbations as instability waves.

Chapters 3–7 are devoted to various generic problems of the theory to show how different isolated factors such as curvature, pressure gradients, flow separation, etc. affect the linear stability of shear flows. The palette of these factors certainly is not exhaustive. However, the set is diverse enough to provide a general view of the basic aspects of the stability analysis.

In Chaps. 8–11, some special but very important topics on linear stability are highlighted. Particularly, various aspects of the initial-value problem with respect to development of small-amplitude disturbances are considered in Chaps. 8–9. The problem of excitation of instability waves by external flow disturbances is considered in Chap. 10. When a linear instability disturbance grows beyond a certain amplitude threshold, it enters the region of its essentially nonlinear, but still deterministic development. The flow modulated by such nonlinear structures is frequently subjected to the phenomenon known as the secondary instability whose initial stage can be described by linear equations of motion. The secondary instability is discussed in Chap. 11.

The nonlinear development of disturbances is accompanied by wave interactions, base-flow distortions, and appearance of turbulent spots. The last part of the book (Chaps. 12–13) is devoted to the description of the main physical mechanisms and phenomena responsible for the final onset of turbulence.

In Appendix, some fundamentals of modern approaches for the prediction and control of the transition are discussed.

Supplementary Material

An interested reader can find details of different aspects of the transition of turbulence in near-wall shear flows in original publications, which are cited in *References* and *Further Reading* sections in the end of each chapter.

We decided to publish the manuscript as an e-book having in mind to make available a set of multimedia files of different value containing supplementary material including video clips and animations.

For those who like to feel the nature through numbers, the book is supplemented by a set of 'toy' MATLAB functions, which allow a reader to understand 'how it works.' The core routines cited in the book are accompanied by interface programs having the same generic names ending with 'Test.' To simplify the process and to give some guidelines, many chapters contain *Exercises*, which can be fulfilled with the help of the functions as a starting point.

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Novosibirsk,
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Nomenclature

Only major symbols are given in the following lists.

A	Amplitude
a	Characteristic height
$\mathbf{c}$	Complex phase velocity
$\mathbf{c}_g$	Complex group velocity
d	Characteristic width
E	Kinetic energy
$\mathbf{e}$	Unit basis vectors
$\mathcal{F}$	Dispersion relation
F	Dimensionless frequency parameter
f	Frequency
G	Growth rate of kinetic energy
$\overline{G}$	Growth rate of kinetic energy scaled with the Reynolds number
H	Shape factor
h	Height
k	Curvature
l	Length
P	Mean pressure
p	Disturbance pressure
R	Radius
$\mathbf{r}$	Radius-vector
T	Temperature
t	Time
$\mathscr{U}$	Full velocity vector
$\mathcal{U}$, $\mathcal{V}$, $\mathcal{W}$	Eulerian components of the full velocity vector
$\mathbf{U}$	Basic flow velocity vector
U, V, W	Eulerian components of the basic flow (mean) velocity vector
$\mathbf{u}$	Disturbance velocity vector
u, v, w	Eulerian components of the fluid disturbance velocity vector
$\overline{U}$	Basic velocity projection to the direction of the wavevector

$\overline{u}$	Disturbance velocity projection to the direction of the wavevector
$\overline{W}$	Basic velocity projection to the direction normal to the wavevector
$\overline{w}$	Disturbance velocity projection to the direction normal to the wavevector
$\mathscr{V}$	Volume of integration

Coordinate systems

x, y, z	Cartesian coordinates (model-fitted)
$\tilde{x}$, y, $\tilde{x}$	Local (external-flow streamline-fitted) Cartesian coordinates
$\bar{x}$, y, $\bar{x}$	Wavevector-fitted Cartesian coordinates
r, θ, z	Cylindrical coordinates

Parameters

De	Dean number
Gö	Görtler number
Re	Reynolds number
Ta	Taylor number
Tu	External flow (free-stream) turbulence level

Greek symbols

Γ	Circulation
Λ	Wavelength parameter
Ψ	Stream function
Ω	Angular velocity
α	Complex streamwise wavenumber
β	Complex transverse wavenumber
β_{H}	Hartree pressure gradient parameter
γ	Angle between κ and streamwise axis
δ	Boundary layer scale
δ^*	Displacement thickness
ε	Small parameter or value
η	Displacement in y
$\hat{\eta}$	Complex amplitude function of normal vorticity mode
$\tilde{\eta}$	Complex amplitude function of normal vorticity initial disturbance
ϑ	Momentum thickness
κ	Wavevector

$\varkappa$ Wavevector length
λ Wavelength
μ Dynamic fluid viscosity
ν Kinematic fluid viscosity
ξ Displacement in x
ρ Fluid density
σ Growth rate
ϕ The angle between the direction of the external-flow streamline and the wedge chord
ζ Displacement in z
ψ Phase variations of a complex amplitude
ω Complex circular frequency

Tensors

S Deformation tensor of the basic flow
s Deformation tensor of the flow disturbances
τ_{ij} Shear stress tensor component

Operators

∇^2 Laplace operator
Δ Difference operator
$\mathcal{D}$ Derivative operator in wall-normal direction
$\mathscr{L}, \mathscr{M}$ Linear operators
$\mathscr{S}$ Bilinear concomitant operator
$\mathscr{P}$ Linear propagator

Accents and superscripts

$\hat{}, \tilde{}, \bar{}$ Refers to complex amplitude function
$'$ Root-mean-square value
$\dagger$ Adjoint quantity
$*$ Complex conjugate
H Hermitian conjugate (complex conjugate and transpose)
T Transpose

Subscripts

0	Refers to initial value
∞	Refers to infinity
2D	Refers to two-dimensional
a	Refers to axial value
ampl	Refers to amplitude
c	Refers to critical layer value
ch	Refers to characteristic value
coat	Refers to coating
E	Refers to critical values of loss of monotonic stability
e	Refers to free-stream (external-flow) value
eff	Refers to effective value
f	Refers to final value
f	Refers to frequency f
i	Refers to imaginary part
L	Refers to critical values of loss of linear stability
G	Refers to critical values of loss of global stability
lam	Refers to laminar regime
m	Refers to mean value
max	Refers to maximum of corresponding value
min	Refers to minimum of corresponding value
NS	Refers to Navier–Stokes equations
OS	Refers to Orr–Sommerfeld equation
p	Refers to particles
r	Refers to real part
s	Refers to inflection point value
Sq	Refers to Squire equation
T	Refers to values of onset of turbulence
x	Refers to the x-coordinate
y	Refers to the y-coordinate
z	Refers to the z-coordinate
r, t, x, y, z, η	Indicate partial derivatives with respect to the corresponding variables
w	Refers to wall
β	Refers to wavenumber β

Acronyms

2D	two-dimensional
3D	three-dimensional
CFI	Crossflow instability
CIFI	Compliance-induced flow instability

DNS	Direct numerical simulation
LNSEs	Locally-nonparallel stability equations
LPSEs	Locally-parallel stability equations
NSEs	Nonparallel stability equations
PIV	Particle image velocimetry
PSEs	Parabolized stability equations
TSI	Tollmien–Schlichting instability