PHENOMENOLOGY AND MATHEMATICS

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Guillermo E. Rosado Haddock is Professor at the University of Puerto Rico at Río Piedras since 1974. He obtained his PhD from the University of Bonn in 1973 with a thesis on Husserl's philosophy of logic and mathematics. Rosado Haddock has published more than thirty-five papers in more than a dozen international journals with prestigious publishers. He is the co-author with Claire Ortiz Hill of a book on Husserl and Frege (Open Court 2000), as well as the author of a book on Frege (Ashgate 2006) and one on Carnap (Ashgate 2008).

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Olav K. Wiegand (Ph.D., Mainz University, 1995) teaches philosophy at the University of Mainz, Germany. He has been a visiting scholar at the M.I.T. (1998) and New York University (1998 and 1999), there working with Kit Fine. His publications include *Interpretationen der Modallogik* (Kluwer, 1998) and articles on Husserl, formal logic, and Kant. Together with R. Dostal, L. Embree, J.N. Mohanty, and J. Kockelmans, he is an editor of a *Festschrift* for Thomas Seebohm entitled *Phenomenology on Kant, German Idealism, Hermeneutics, and Logic* (Kluwer, 2000). Since 2005, he has worked as healthsystem consultant for a private company, but he still also teaches at Mainz University.

LIST OF ABBREVIATIONS

APS *Analyses Concerning Passive and Active Synthesis: Lectures on Transcendental Logic.* Translated by Anthony J. Steinbock. Kluwer: Dordrecht, Boston, London. 2001.

CM English translation of Hua 1: *Cartesian Meditations*. Translated by Dorion Cairns. The Hague: Nijhoff. 1970.

Crisis English translation of Hua 6: *The Crisis of European Sciences and Transcendental Phenomenology*. Translated by David Carr. Evanston: Northwestern University Press. 1970.

EU English translation of *Erfahrung und Urteil* [1939]: *Experience and Judgment.* Translated by J. Churchill and K. Ameriks. Evanston: Northwestern University Press. 1973.

FTL English translation of Hua 17: *Formal and Transcendental Logic*. Translated by Dorion Cairns. The Hague: Martinus Nijhoff, 1978.

Ideas I English translation of Hua 3: *Ideas*, *General Introduction to Pure Phenomenology*. Translated by Boyce Gibson, New York: Collier Books. 1962.

LI 1-6 English translation of Hua 19: *Logical Investigations* Vols. 1–2. Translated by J. N. Findlay. London: Routledge & Kegan Paul, 1970.

PA English translation of Hua 12: *Philosophy of Arithmetic, Psychological and Logical Investigations with Supplementary Texts from 1887–1901*. Translated by Dallas Willard, Dordrecht: Kluwer, 2003.

Prolegomena English translation of Hua 18: *Logical Investigations* 1. Translated by J. N. Findlay. London: Routledge & Kegan Paul, 1970.

Husserliana volumes referred to

Hua 1 *Cartesianische Meditationen und Pariser Vorträge.* [Cartesian meditations and the Paris lectures.] Edited by S. Strasser. The Hague, Netherlands: Martinus Nijhoff, 1973.

Hua 3 *Ideen zu einer reinen Phänomenologie und phänomenologischen Philosophie.* Erstes Buch: Allgemeine Einführung in die reine Phänomenologie. [Ideas: general introduction to pure phenomenology and to a phenomenological philosophy. First book.] Edited by Walter Biemel. The Hague, Netherlands: Martinus Nijhoff Publishers, 1950.

Hua 7 *Erste Philosophie (1923/4).* Erste Teil: Kritische Ideengeschichte. [First philosophy (1923/24). First part: the critical history of ideas.] Edited by Rudolf Boehm. The Hague, Netherlands: Martinus Nijhoff, 1956.

Hua 9 *Phänomenologische Psychologie.* Vorlesungen Sommersemester. 1925. [Phenomenological psychology. Lectures from the summer semester. 1925.] Edited by Walter Biemel. The Hague, Netherlands: Martinus Nijhoff, 1968.

Hua 10 *Zur Phänomenologie des inneren Zeitbewusstseins (1893–1917).* [The Phenomenology of internal time-consciousness (1893–1917).] Edited by Rudolf Boehm. The Hague, Netherlands: Martinus Nijhoff, 1969.

Hua 12 *Philosophie der Arithmetik.* Mit ergänzenden Texten (1890–1901). [Philosophy of arithmetic. With complementary texts. (1890–1901).] Edited by Lothar Eley. The Hague, Netherlands: Martinus Nijhoff, 1970.

Hua 18 *Logische Untersuchungen.* Erster Teil. Prolegomena zur reinen Logik. Text der 1. und der 2. Auflage. [Logical investigations: first part. Prolegomena to pure logic. Text of the first and second edition.] Halle: 1900, rev. ed. 1913. Edited by Elmar Holenstein. The Hague, Netherlands: Martinus Nijhoff, 1975.

Hua 19 *Logische Untersuchungen.* Zweiter Teil. Untersuchungen zur Phänomenologie und Theorie der Erkenntnis. In zwei Bänden. [Logical investigations. Second part. Investigations concerning phenomenology and the theory of knowledge. In two volumes.] Edited by Ursula Panzer. Halle: 1901; rev. ed. 1922. The Hague, Netherlands: Martinus Nijhoff, 1984.

Hua 20/1 *Logische Untersuchungen. Ergänzungsband.* Erster Teil. Entwürfe zur Umarbeitung der VI. Untersuchung und zur Vorrede für die Neuauflage der Logischen Untersuchungen (Sommer 1913). [Logical investigations. Supplementary volume. Draft plan for the revision of the 6th Logical Investigation and the foreword of the Logical Investigations (Summer 1913).] Edited by Ulrich Melle. The Hague, Netherlands: Kluwer Academic Publishers, 2002.

Hua 21 *Studien zur Arithmetik und Geometrie.* Texte aus dem Nachlass (1886–1901). [Studies on arithmetic and geometry. Texts from the estate (1886–1901)]. Edited by Ingeborg Strohmeyer. The Hague, Netherlands: Martinus Nijhoff, 1983.

Hua 22 *Aufsätze und Rezensionen (1890–1910).* [Articles/essays and reviews (1890–1910).] Edited by B. Rang. The Hague, Netherlands: Martinus Nijhoff, 1979.

Hua 24 *Einleitung in die Logik und Erkenntnistheorie.* Vorlesungen 1906/07. [Introduction to logic and the theory of knowledge. Lectures 1906/07]. Edited by Ullrich Melle. The Hague, Netherlands: Martinus Nijhoff, 1985.

Hua 26 *Vorlesungen über Bedeutungslehre.* Sommersemester 1908. [Lectures on the doctrine of meaning: summer semester 1908.] Edited by Ursula Panzer. The Hague, Netherlands: Martinus Nijhoff, 1987.

Hua 28 *Vorlesungen über Ethik und Wertlehre.* 1908–1914. [Lectures on ethics and value theory, 1908–1914.] Edited by Ullrich Melle. The Hague, Netherlands: Kluwer Academic Publishers, 1988.

Hua 30 *Logik und allgemeine Wissenschaftstheorie.* Vorlesungen 1917/18. Mit ergänzenden Texten aus der ersten Fassung 1910/11. [Logic and general theory of science. Lectures 1917/18, with complementary texts from the first version 1910/11.] Edited by Ursula Panzer. The Hague, Netherlands: Kluwer Academic Publishers, 1995.

Briefwechsel 1–10 Husserl, Edmund. 1994. *Briefwechsel*. [Correspondence.] Edited by Karl Schuhmann. The Hague, Netherlands: Kluwer Academic Publishers.

INTRODUCTION

It is beginning to be a commonplace that Edmund Husserl (1859–1938), the founder of the phenomenological movement, was originally a mathematician who studied with Weierstrass and Kronecker. The roots of the phenomenological tradition are in the nineteenth century mathematics and logic, very much like those of the analytic tradition. As analytic philosophy has grown to view itself as a historically conditioned tradition the relationship between Husserl and other nineteenth century logicians and mathematicians have become a focus of much research. An early pioneer was Dagfinn Føllesdal's classic (1958) that appeared in English in Leila Haaparanta's (1994) collection of essays *Mind, Meaning, and Mathematics*. In it Føllesdal suggested that Frege possibly influenced Husserl to turn away from psychologism. The paper initiated a debate that has continued ever since (e.g., Rosado-Haddock 1973 and together with Claire Hill 2000, J.N. Mohanty 1974 and 1982, see also Chapters 2 and 3 in the present volume).

The publication of thousands of pages of Husserl's writings has been another important factor shaping the research on Husserl. Indeed, of 40 *Husserliana* volumes that had appeared by 2005, 20 has appeared after 1980 and 9 after 2000, the whole *Dokumente* series has appeared after 1980, and the whole *Materialien* series has appeared after 2000. Much of the newly published texts are about mathematics and logic. In particular, almost all of early Husserl's writings on mathematics are available to the general public in *Husserliana* volumes 21 (1983) on arithmetics and geometry, 22 (*Aufsätze und Rezensionen*), and of course 12 (*Philosophy of Arithmetic*). Thanks to Dallas Willard's monumental work, the Husserliana volumes 12 and 21 have appeared also in English as the Collected Works 5 (1994) and 10 (2003). Likewise Husserl's lectures on logic from various years published in the *Materialien* series document the development of Husserl's views on logic from 1896 onwards. The vast amounts of new material available has deepened the scholarship considerably and has been the focus of for example Hill (1991, 2000), Tieszen (1989, 2005), Seebohm (1991), and Lohmar (1989, 2000). Arguably, the growing amount of historical research on Husserl's development has paved the way for overcoming the juxtaposition between the analytic and continental traditions.

Presumably also at least partly due to the interest in the common roots of phenomenological and analytical traditions, a growing amount of research has focused on the development of modern logic, mathematics and physics in the nineteenth and the early twentieth century producing books such as *Die Philosophie und die Mathematik: Oskar Becker in der mathematischen Grundlagendiskussion* (2005) edited by Volker Peckhaus, *The Architecture of Modern Mathematics* (2006) by Jeremy Gray and Jose Ferreiros, *Intuition and the Axiomatic Method* (2006) edited by Emily Carson and Renate Huber, the forthcoming edition of David Hilbert's lectures on the foundations of mathematics and physics 1891–1933 by Michael Hallett and Ulrich Majer (Volume 1 has appeared when writing this introduction), and *The Development of Modern Logic* (2009), edited by Leila Haaparanta. In these works the development of mathematics and logic during Husserl's time is discussed and interest to Husserl's possible contribution to it is shown. Moreover, there are attempts to include discussion of Husserl to the history of the late nineteenth century logic. For example, Gabbay's *Handbook on History of Logic* (2004) has an extensive chapter on Husserl's logic written by Richard Tieszen, not to mention the special issue of *Philosophia Mathematica* on phenomenology and mathematics that appeared in June 2002.

The above mentioned historical approaches to Husserl and mathematics focus particularly on Husserl's early writings on mathematics and logic. Another, more systematic approach comes from the attempts to situate also later Husserl's thoughts on mathematics in the general *Grundlagenstreit* of the early twentieth century discussions on the foundations of mathematics. At this time Husserl's own writings focused on more general questions in phenomenology and his writings on mathematics and logic were rather suggestive. Moreover, Husserl's manuscript A I 35 that is arguably one of the most important texts on mathematics in Husserl's later works and is referred to several times in the present volume is only now being published. Husserlian legacy continued mainly, in more or less faithful way, in the work of Oskar Becker and Hermann Weyl. Thus many studies related to the phenomenology of mathematics focus rather on these figures than on Husserl himself (e.g., Mancosu and Ryckman 2002, 2005). The comparisons of Husserl' s approach to the intuitionists, Brouwer (van Atten), in particular, are closely related to these works as well as now already rather numerous investigations discussing Gödel's background in Husserl (Føllesdal, Van Atten, Kennedy). Upon publication of Heinz-Dieter Ebbinghaus's book on Ernst Zermelo (2007), one hopes to read papers elaborating on Husserl's relationship to Zermelo soon too.

As becomes clear from the above, Husserl's views in relation to history and philosophy of mathematics and logic provide us an incredibly rich field for research. The *Husserliana* volumes offer enormously material for internal studies on Husserl's development for also those working outside the archives. Husserl wrote at the time when modern logic and mathematics were rapidly developing toward their current outlook. Thus his writings can also be fruitfully compared and contrasted with both nineteenth century figures such as Boole, Schröder and Weierstrass as well as the twentieth century characters like Heyting, Zermelo, and Gödel. Besides the more historical studies, both the internal ones on Husserl alone and the external ones attempting to clarify his role in the more general context of the developing mathematics and logic, the field has also systematic importance. Indeed, one motivation of the present volume is to make sense of Husserl's transcendental idealism in mathematics.

The volume at hand manifests all the above mentioned aspects in which Husserl's views on mathematics are of interest and can be studied. It gathers the contributions of the main scholars of the field into one publication for the first time. Thus it gives an overview of the current debates and themes in the phenomenology of mathematics. The systematic and historical approaches are intertwined in the contributions. Ultimately, the papers chart answers to the question "What kind of philosophy of mathematics is phenomenology?" In the course of answering this question Husserl's philosophy of mathematics is compared and contrasted to the constructivist as well as various kinds of Platonist views of mathematics.

In Chapter 1 "Mathematical Realism and Transcendental Phenomenological Idealism" Richard Tieszen addresses the question whether mathematical realism is compatible with transcendental phenomenological idealism. His answer is that the views are indeed compatible provided that neither "idealism" nor "realism" are understood in their naïve senses. Rather they should be understood in accordance to Husserl's transcendental phenomenology in which a kind of Platonism is embedded within transcendental idealism. Tieszen calls the view "constituted Platonism" or "constituted realism." Tieszen first considers the standard simple formulations of realism and idealism (anti-realism) about mathematics. Mathematical realism is the view that there are mind-independent abstract (or "ideal") mathematical objects or truths; the standard antirealist view is the negation of this view. To Tieszen Husserl's view about mathematical objects is Platonist rather than Aristotelian realist. Tieszen then goes on to consider transcendental phenomenological idealism. With phenomenological reduction we can accomplish the point of view of reflection and focus on how objects are given to us. Now we can find out that the ideal objects such as objects of mathematics are constituted as transcendent objects, which explains the choice of his term "constituted Platonism." Tieszen then distinguishes several conceptions of how objects can be considered mind-dependent and mind-independent or immanent and transcendent. Naive mind-independence is metaphysical Platonism. But within the mind-dependent sphere there is also mind-independence, the constituted realism of Husserl. In the end Tieszen raises interesting questions about which mathematical objects are constituted as real as well as a question about the compatibility of Husserl's view with Putnam's internal realism.

The next two chapters purport to demonstrate that Husserl was not Brouwerian intuitionist, nor constructivist of any sort. In his "Platonism, Phenomenology and Interderivability," Guillermo Rosado Haddock defends the view that Husserl is a Platonist rather than a constructivist philosopher. Van Atten has held that this is not so obvious when Husserl's later texts are taken into consideration. Rosado Haddock, on basis of Husserl's manuscript A I 35 from the years 1912 and 1920, aligns Husserl with Cantor and Zermelo. According to him, here, in *Formale und transzendentale Logik* (1929) as well as in his lectures from 1906–1907 and of 1917–1918, Husserl propounds essentially the same Platonist philosophy of mathematics as in his earlier texts. Rosado Haddock then examines the interderivability phenomena, i.e., that equivalent mathematical statements can be found in seemingly unrelated parts of mathematics, arguing that while Husserl's Platonism is able to tackle the issue, a constructivist has difficulties in accounting for it. Rosado Haddock then discusses Husserl's concept of situations of affairs (*Sachlage*) as distinguished from his notion of states of affairs (*Sachverhalt*). With this distinction Husserl is thus able to assess the interderivability phenomena (also contrary to Frege). The outcome is that while e.g., the Axiom of Choice and Tychonoff's Theorem refer to different states of affairs, their situation of affairs remains the same.

In her "Husserl on Axiomatization and Arithmetic" Claire Ortiz Hill complements Rosado Haddock's chapter by a more systematic demonstration of the extent to which Husserl rejected Brouwer's intuitionism. She draws from the newly published Husserl's logic courses from 1896 and 1902/03. She explores various aspects of the axiomatic nature of Husserl's logic, ultimately arguing that Husserl's approach is much closer to that of David Hilbert than Brouwer's intuitionism. In so doing, she touches upon Husserl's view of the relationship between mathematics and logic, arguments against psychologism, objectivity of meaning, the law of the excluded middle, axiomatic account of number, Husserl's account of time, Husserl's view of the three levels of logic and the theory of manifolds, and the relationship between mathematics and phenomenology. In all of these respects Husserl's views differ from those of Brouwer. She concludes her contribution raising the question about the systematic importance of Husserl's views, which, she thinks, should be given a try next, once the relationship of his ideas to Frege's, Brouwer's, and Hilbert's theories has been clarified.

Dieter Lohmar's chapter (Chapter 4) then focuses on Husserl's notion of categorial intuition emphasizing the empirical side of Husserl's approach. Dieter Lohmar's paper focuses on Husserl's notion of intuition and evidence in formal contexts. In mathematics we can gain the highest form of evidence which is apodictic evidence. This takes place by means of what Husserl calls *Wesensschau*, which is a special case of categorial intuition. Lohmar's aim is to explain how apodictic evidence is gained in mathematics by means of the method of eidetic variation.

Lohmar starts with an analysis of the sensible givenness of objects, noting that already the sensual perception of an object exceeds what is actually given by our senses. Intentions such as "This book is green" are fulfilled not only with sensuality but also something more that relates back to our thinking activity, namely "synthesis of coincidence." Mathematical knowledge is in most cases independent of sensuality but nevertheless intuitive. Lohmar then goes on to discuss specifically mathematical knowledge where the syntheses of coincidence are more easily structured and more distinct due to the lack of horizonal intentions, which usually accompany the intending of everyday objects. Lohmar then goes on to explain the method of seeing essences, i.e., *Wesensschau* with which a priori insight into the universal structures of consciousness, sounds, colors, as well as, geometry, arithmetic and other parts of mathematics can be obtained. Lohmar summarizes the development of the method in Husserl's texts and settles to detail it on basis of Husserl's final form of the method worked out in the lecture *Phenomenological Psychology*. In it an element of sense that I can freely go on with the variation is added to the process of variation idealizing it so that it can be termed an "infinite variation." Lohmar goes on to explain in detail various stages in the method of eidetic variation, with which phenomenological a priori is obtained. Lohmar then discusses several examples showing how apodictic evidence can be obtained first in material mathematical disciplines and then in formal axiomatic contexts.

In Chapter 5 Jaakko Hintikka compares Husserl's views to e.g., Mach, Russell, Wittgenstein, and Gödel, and points out the importance of Husserl's view of the theory of theories. Hintikka starts his contribution with a discussion of what phenomenology is and settles to the sense of the term that derives from Mach and Hering, and, according to Hintikka, is the sense in which Husserl himself uses it too. Accordingly Hintikka finds Husserl's and Mach's approaches rather similar disregarding their views of mathematics. While mathematics to Mach is tautological, Husserl sought for a much richer approach to mathematics. Hintikka examines Husserl's notion of *Anschauung*, which Hintikka takes Husserl to understand in a minimal sense as "immediate knowledge," a counterpart to Russellian acquaintance. Husserl developed this into *Wesensschau*, which provides us with an access to what Husserl calls "formal ontology" as well as various "regional ontologies." Hintikka compares Husserl's view to Aristotle, and then to Wittgenstein, whom Hintikka takes to be closer to Husserl than is usually thought. Wittgenstein's later criticism of Husserl, Hintikka views as directed at expressibility of the testimony of *Wesensschau*, and not at the possibility of a kind of *Wesensschau*. This is the source for the impossibility of a genuine theory of logical forms to Wittgenstein. Hintikka then uses Husserl's Aristotelianism to explain why Husserl is not a finitist or an intuitionist as Husserl held that the human mind is able to grasp directly abstract structures. Behind this view is Husserl's vision of a universal "structure of all structures" or "model of all models," which Hintikka suggests was tacitly in the minds of many mathematicians contemporary to Husserl. While the comprehensive ideas such as the set of all sets have turned out to be difficult to implement, Husserl's view of logic still lives in the model theory. Hintikka concludes his discussion with a brief remark on Gödel. The comparison between Husserl and Gödel proves favorable for Husserl especially in philosophical respects.

Mirja Hartimo's contribution "Development of mathematics and the birth of phenomenology" ties Husserl's view to the more general development of mathematics in the late nineteenth century. Her aim is to connect Husserl's discovery of categorial intuition to his investigations into the development of mathematics in the late nineteenth century. She focuses in particular on Husserl's Weierstrassian heritage, which shows in Husserl's search for intuitively evident foundations for the basic concepts of mathematics. Following the mainstream view of mathematics Husserl adopts a structural, axiomatic view of mathematics by the turn of the century. Her view of Husserl's term *Definitheit* is that it means roughly the categoricity of an axiomatic system, i.e., that the axioms define the formal domain uniquely, up to isomorphism. However, contrary to Hilbert, Husserl remains Weierstrassian in that he continues to demand intuitively evident foundations for axiomatics. To that extent Husserl developed the notion of categorial intuition. Hartimo then goes on to discuss the consequences of the view to Husserl's approach toward reality. She distinguishes two senses in which Husserl can be said to be a Platonist. The other derives from Lotze and relates to Husserl's objectivist view of the formal objects. The other derives originally from Weiersrass and relates to the search for *justification* for the axiomatic systems.

In his chapter "Beyond Leibniz: Husserl's Vindication of Symbolic Knowledge," Jairo José da Silva gives another account of the development of Husserl's views of symbolic knowledge in mathematics up to his *Doppelvortrag* in 1901. He focuses on how Husserl struggled with the problem of how to explain that we can obtain knowledge by operating "blindly" with symbols according to rules, even when these symbols do not represent anything. The imaginaries are improper representations since they do not represent any object. Nevertheless in calculations they are needed and they pass off as denoting something.

Da Silva follows Husserl's development from his first discussion of the problem in the *Philosophy of Arithmetic* to his Göttingen talks. In the *Philosophy of Arithmetic* the problem had two variants, one concerning justification for arithmetical computations, the other is about the symbols 0 and 1. Husserl's solution to the former problem is that the symbolic system and its isomorphic copy, the system of number concepts, share a common formal structure. Husserl's reason for accepting 0 and 1 as numbers, according to da Silva, is that they are required as necessary completions of arithmetical domains. Husserl's 1891 review of Schröder's *Lectures on the Algebra of Logic* already anticipates Husserl's mature view, which he discusses in his Göttingen talks in 1901. In these lectures Husserl held that the introduction of imaginary elements in a domain is allowed provided: (1) the (formal) theory extending the (formal) theory of the domain in question by means of formal axioms introducing imaginaries in an extended language is consistent, and (2) the formal theory of the domain, written in the restricted language without imaginaries, is complete, in Husserl's terms, definite. The solution tells that imaginary entities can be treated like the real ones. Nonetheless, according to *Logical Investigations* symbolic theories are mere forms of theories. The creation and study of formal theories for their own sake, would amount to a "formalist alienation." Hence Da Silva suggests that Husserl's claim could be understood to be that formal theories are only interesting if they can be applied, thus emphasizing more instrumentalist or pragmatist aspects of Husserl's views. Imaginaries are useful as they enrich the structural milieu so that the problems can be better solved.

The last two chapters elaborate specifically on the systematic value of Husserl's philosophy and relate it to the more general present day discussions, first in philosophy of mathematics, second in metaphysics. In his "Mathematical Truth Regained," Robert Hanna seeks a fully realistic and inescapably anthropocentric conception of mathematical truth and knowledge—*real mathematics for humans*, as he puts it. In so doing, he offers what he calls "a positive Kantian phenomenological solution" to Benacerraf's Dilemma (BD) of reconciling a "standard" (i.e., classical Platonic) semantics with a reasonable epistemology of mathematical knowledge. Hanna's solution is positive because it accepts Benacerraf's preliminary philosophical assumptions about the nature of semantics and knowledge, as well as all the basic premises of BD, and then shows how we can still reject the skeptical conclusion BD and adequately explain mathematical knowledge. The solution relies on mathematical structuralism, i.e., on a view that mathematical entities are not ontologically autonomous or independent objects, but instead are essentially positions-in-a-mathematical-structure (a view not necessarily so far from Husserl's conception either). In particular, Hanna interprets mathematical objects with the role players of the roles determined by the system as a whole. However, Hanna favors specifically Kantian Structuralism which is a *non-reductive* and *ante rem* version of structuralism in contrast to Benacerraf's structuralism, which is reductive and *ante rem*. In Hanna's favorite kind of structuralism time-structure is what binds arithmetic to our world. With this view Hanna solves the BD. To Hanna, Husserl's views prove useful in explaining how mathematical knowledge is possible. He proposes what he calls the Husserl-Wittgenstein Theory of Logical and Mathematical Self-Evidence (the HW theory). The HW theory is based on Husserl's doctrine of categorial intuition and related views in Wittgenstein's *Tractatus*, according to which abstract structures are immediately represented in our non-conceptual conscious awareness.

The volume ends with Olav Wiegand's "On Referring to Gestalts," in which he explores mereological semantics on the basis of Husserl's phenomenology and Gestalt psychology leaning on the work of Aron Gurwitsch. The primary motivation of the paper is to formalize the notion of structured whole. In his paper, Wiegand defines Gestalts as "R-structured wholes" aiming to capture the interconnectedness of all parts of a Gestalt. He then discusses relations from the point of view of mereological semantics. In so doing Wiegand shows the usefulness of Husserl's views to contemporary formal semantics and metaphysics.