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Theory of Gravitational Interactions

Springer

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ISSN 2192-4791 ISSN 2192-4805 (electronic)
Undergraduate Lecture Notes in Physics
ISBN 978-88-470-2690-2 ISBN 978-88-470-2691-9 (eBook)
DOI 10.1007/978-88-470-2691-9
Springer Milan Heidelberg New York Dordrecht London

Library of Congress Control Number: 2013930607

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To my parents

Preface

This book grew out of lectures given by the author at the University of Turin and at the University of Bari. It is primarily intended for undergraduate students taking classes in gravitational theory, as prescribed by modern academic plans to graduate in Physics with a theoretical/high-energy physics or astrophysics curriculum. The challenge is to provide students with a textbook which, on one hand, can represent a self-contained reference for a semester cycle of lectures and, on the other hand, may be accessible and of profitable use also for students having different interests and following different academic tracks.

To this aim the book includes a first, conventional part introducing general relativity as a geometric theory of the macroscopic gravitational field, and a second, more advanced part, connecting general relativity to the gauge theories of fundamental interactions. A discussion of the deep analogies (and of the physical differences) existing between gravity and the other standard-model interactions fills a gap which is present within the traditional geometric approach to general relativity, and which usually puzzles students about the role of gravity in the context of a unified model of all interactions.

In this spirit, the formalism of differential geometry has been reduced to the necessary minimum, leaving more room to current interesting aspects of gravitational physics of both applicative/observational type (such as the phenomenology of gravitational waves) and theoretical/fundamental type (such as the gravitational interactions of spinors, supergravity and higher-dimensional gravity). We have included, however, a final appendix introducing the so-called "Cartan calculus" of exterior (or differential) forms, in view of the important applications of this formalism not only to the gravitational theory but also to many other fields of theoretical physics. A second appendix introduces various possible approaches to the problem of embedding a four-dimensional theory of gravity in the context of a higher-dimensional space–time manifold.

For most profitable use of this book the reader is expected to have a basic knowledge of special relativity, electromagnetic theory and classical theory of fields. Except for the above input, however, the book aims at being self-contained as much as possible, following the informal style of class lectures where all the required notions and techniques are explicitly recalled and/or introduced whenever necessary. Also, for a better pedagogic efficiency, all computations are explicitly carried out in the main text (leaving no "voids" to be filled by the readers), or presented as solved exercises at the end of each chapter.

The present book is certainly not intended to represent a complete reference for a rigorous and comprehensive study of all theoretical aspects of the gravitational interaction. Its main purpose is to provide students with the basic starting notions, enabling them to do further independent work and subsequent deeper studies on more professional textbooks and papers. The readers interested in advanced discussions of some specific topic are strongly advised to refer to the list of specialistic books presented in the bibliography.

Finally, it should be noted that this book deliberately avoids any gravitational application to cosmology and large-scale astrophysics, because—according to modern academic plans of studies—they are a matter of specific courses and lectures, well separated from a course on the theory gravity. The field of relativistic cosmology is today so extended, with so many branches and applications, as to deserve by itself a dedicated book. We refer, for this purpose, to the excellent books quoted in the bibliography, as well as to an introduction to theoretical cosmology which represents the natural continuation of this book, and which currently exists as a Springer Italian edition [20].

Acknowledgements

It is a pleasure, as well as a duty, to thank all my colleagues and students for their comments, suggestions, and criticism that contributed over the years to correct and improve these lecture notes. Listing all of them would be an impossible task, so let me thank them jointly for their important help.

A warm acknowledgement is also due to Venzo De Sabbata, who was one of my Professors when (many years ago!) I was a student of Physics at the University of Bologna. Professor De Sabbata introduced me to the study of gravitation and cosmology, and the interest he was able to stimulate towards those branches of physics was so intense as to be still alive, and fully effective, even today in my present scientific activity.

Finally, I wish to thank Marina Forlizzi, Executive Editor for Springer-Verlag, for her kind encouragement, advice, and many useful suggestions.

Cesena, Italy Maurizio Gasperini

Notations, Units and Conventions

Throughout this book we will use the index 0 for the time-like components of vector and tensor objects, while the indices $1, 2, 3$ will refer to the space-like components. For the space–time metric $g_{\mu\nu}$ we will adopt the signature with a positive time-like eigenvalue, namely:

$$g_{\mu\nu} = \mathrm{diag}(+, -, -, -).$$

Our conventions for the curvature and the covariant derivatives will be as follows. Riemann tensor:

$$R_{\mu\nu\alpha}{}^{\beta} = \partial_\mu \Gamma_{\nu\alpha}{}^{\beta} + \Gamma_{\mu\rho}{}^{\beta} \Gamma_{\nu\alpha}{}^{\rho} - \{\mu \leftrightarrow \nu\},$$

where the symbol $\{\mu \leftrightarrow \nu\}$ means that we must insert all the preceding terms with the indices μ and ν interchanged between themselves. Ricci tensor:

$$R_{\nu\alpha} = R_{\mu\nu\alpha}{}^{\mu}.$$

Covariant derivative:

$$\nabla_\mu V^\alpha = \partial_\mu V^\alpha + \Gamma_{\mu\beta}{}^{\alpha} V^\beta; \qquad \nabla_\mu V_\alpha = \partial_\mu V_\alpha - \Gamma_{\mu\alpha}{}^{\beta} V_\beta;$$

Lorentz covariant derivative:

$$D_\mu V^a = \partial_\mu V^a + \omega_\mu{}^a{}_b V^b; \qquad D_\mu V_a = \partial_\mu V_a - \omega_\mu{}^b{}_a V_b.$$

Also, the symbol $\Box$ will denote the usual d'Alembert operator of the flat Minkowski space–time, i.e.:

$$\Box = \eta^{\mu\nu} \partial_\mu \partial_\nu = \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2,$$

where η is the Minkowski metric and $\nabla^2 = \delta^{ij} \partial_i \partial_j$ the Laplacian operator of the Euclidean three-dimensional space.

Unless otherwise stated, we will use small Latin letters $i, j, k, \ldots$ for the spatial indices $1, 2, 3$; small Greek letters $\mu, \nu, \alpha, \ldots$ for the space–time indices $0, 1, 2, 3$. In higher-dimensional space–times, with a number $d > 3$ of spatial dimensions, the space–time indices will be denoted instead by capital Latin letters: $A, B, C, \ldots = 0, 1, 2, 3, \ldots, d$.

Two (or more) indices, when enclosed in round or square brackets, will satisfy, respectively, the symmetry or antisymmetry property defined by:

$$T_{(\alpha\beta)} \equiv \frac{1}{2}(T_{\alpha\beta} + T_{\beta\alpha}), \qquad T_{[\alpha\beta]} \equiv \frac{1}{2}(T_{\alpha\beta} - T_{\beta\alpha}).$$

In the presence of more than two indices, two symmetric (or antisymmetric) indices which are not contiguous will be separated from the others by a vertical bar. We write, for instance:

$$T_{(\mu|\alpha|\nu)} \equiv \frac{1}{2}(T_{\mu\alpha\nu} + T_{\nu\alpha\mu}),$$
$$T_{[\mu|\alpha|\nu]\beta} \equiv \frac{1}{2}(T_{\mu\alpha\nu\beta} - T_{\nu\alpha\mu\beta}),$$

to mean that the first object is symmetric with respect to μ and ν (at fixed α), while the second object is antisymmetric with respect to μ and ν (at fixed α and β). An so on for a higher number of indices.

Such a symmetrization/antisymmetrization procedure can be easily extended to an arbitrary number of indices $n \geq 2$, by including all their possible permutations and then dividing by the total number of permutations, $n!$. For the symmetrization procedure all permutations are to be added with the same sign, while, for the antisymmetrization procedure, even permutations are to be added with the plus sign and odd permutations with the minus sign. For instance:

$$T_{(\mu\nu\alpha)} = \frac{1}{3!}(T_{\mu\nu\alpha} + T_{\nu\alpha\mu} + T_{\alpha\mu\nu} + T_{\mu\alpha\nu} + T_{\nu\mu\alpha} + T_{\alpha\nu\mu}),$$
$$T_{[\mu\nu\alpha]} = \frac{1}{3!}(T_{\mu\nu\alpha} + T_{\nu\alpha\mu} + T_{\alpha\mu\nu} - T_{\mu\alpha\nu} - T_{\nu\mu\alpha} - T_{\alpha\nu\mu}).$$

An so on for a higher number of indices. Finally, the fully antisymmetric tensor (also called the Levi-Civita symbol) of the Minkowski space–time, $\epsilon^{\mu\nu\alpha\beta}$, will be defined according to the following conventions:

$$\epsilon^{0123} = +1, \qquad \epsilon_{\mu\nu\alpha\beta} = -\epsilon^{\mu\nu\alpha\beta}.$$

Its generalization to the case of an arbitrary Riemann manifold will be introduced in Chap. 3, Sect. 3.2.

The units that will be adopted for our numerical estimates and for the electromagnetic variables are the so-called (unrationalized) c.g.s. units, where the Maxwell equations take the form:

$$\partial_\mu F^{\mu\nu} = \frac{4\pi}{c} J^\nu,$$
$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \qquad A^\mu = (\phi, \boldsymbol{A}).$$

When dealing with scalar and spinor fields we will instead refer to the more convenient "natural" units, where both the light velocity c and the Planck constant $\hbar$ are set to one, $c = \hbar = 1$. In such units the Newton constant G acquires length-squared (or inverse mass-squared) dimensions, and is related to the Planck mass M_P and to the Planck length λ_P by:

$$G^{-1} = M_\mathrm{P}^2 = \lambda_\mathrm{P}^{-2}.$$

In c.g.s. units:

$$M_\mathrm{P} = \left(\frac{\hbar c}{G}\right)^{1/2} \simeq 2 \times 10^{-5}\ \mathrm{g},$$

$$\lambda_\mathrm{P} = \left(\frac{G\hbar}{c^3}\right)^{1/2} \simeq 1.6 \times 10^{-33}\ \mathrm{cm}.$$

The energy associated to the Planck mass is $E_\mathrm{P} = M_\mathrm{P} c^2 \simeq 10^{19}$ GeV, where 1 GeV $= 10^9$ eV is the energy scale associated to the mass of the proton. As evident from the above definitions, the Planck energy scale characterizes the strength of the gravitational coupling, and also controls the importance of the quantum corrections to the equations of the classical gravitational theory.

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