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Information Geometry and Its Applications

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Preface

Information geometry is a method of exploring the world of information by means of modern geometry. Theories of information have so far been studied mostly by using algebraic, logical, analytical, and probabilistic methods. Since geometry studies mutual relations between elements such as distance and curvature, it should provide the information sciences with powerful tools.

Information geometry has emerged from studies of invariant geometrical structure involved in statistical inference. It defines a Riemannian metric together with dually coupled affine connections in a manifold of probability distributions. These structures play important roles not only in statistical inference but also in wider areas of information sciences, such as machine learning, signal processing, optimization, and even neuroscience, not to mention mathematics and physics.

It is intended that the present monograph will give an introduction to information geometry and an overview of wide areas of application. For this purpose, Part I begins with a divergence function in a manifold. We then show that this provides the manifold with a dually flat structure equipped with a Riemannian metric. A highlight is a generalized Pythagorean theorem in a dually flat information manifold. The results are understandable without knowledge of differential geometry.

Part II gives an introduction to modern differential geometry without tears. We try to present concepts in a way which is intuitively understandable, not sticking to rigorous mathematics. Throughout the monograph, we do not pursue a rigorous mathematical basis but rather develop a framework which gives practically useful and understandable descriptions.

Part III is devoted to statistical inference, where various topics will be found, including the Neyman–Scott problem, semiparametric models, and the EM algorithm. Part IV overviews various applications of information geometry in the fields of machine learning, signal processing, and others.

Allow me to review my own personal history in information geometry. It was in 1958, when I was a graduate student on a master's course, that I followed a seminar on statistics. The text was "Information Theory and Statistics" by S. Kullback, and

a professor suggested to me that the Fisher information might be regarded as a Riemannian metric. I calculated the Riemannian metric and curvature of the manifold of Gaussian distributions and found that it is a manifold of constant curvature, which is no different from the famous Poincaré half-plane in non-Euclidean geometry. I was enchanted by its beauty. I believed that a beautiful structure must have important practical significance, but I was not able to pursue its consequences further.

Fifteen years later, I was stimulated by a paper by Prof. B. Efron and accompanying discussions by Prof. A.P. Dawid, and restarted my investigation into information geometry. Later, I found that Prof. N.N. Chentsov had developed a theory along similar lines. I was lucky that Sir D. Cox noticed my approach and organized an international workshop on information geometry in 1984, in which many active statisticians participated. This was a good start for information geometry.

Now information geometry has been developed worldwide and many symposia and workshops have been organized around the world. Its areas of application have been enlarged from statistical inference to wider fields of information sciences.

To my regret, I have not been able to introduce many excellent works by other researchers around the world. For example, I have not been able to touch upon quantum information geometry. Also I have not been able to refer to many important works, because of my limited capability.

Last but not least, I would like to thank Dr. M. Kumon and Prof. H. Nagaoka, who collaborated in the early period of the infancy of information geometry. I also thank the many researchers who have supported me in the process of construction of information geometry, Profs. D. Cox, C.R. Rao, O. Barndorff-Nielsen, S. Lauritzen, B. Efron, A.P. Dawid, K. Takeuchi, and the late N.N. Chentsov, among many many others. Finally, I would like to thank Ms. Emi Namioka who arranged my handwritten manuscripts in the beautiful TEX form. Without her devotion, the monograph would not have appeared.

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