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Shuheï Mano

# Partitions, Hypergeometric Systems, and Dirichlet Processes in Statistics

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# Preface

This monograph is on statistical inferences related to some combinatorial stochastic processes. Especially, it discusses the intersection of three subjects that are studied almost independently from each other: partitions, hypergeometric systems, and Dirichlet processes. It is shown that the three subjects involve a common structure called exchangeability. Then, inference methods based on their algebraic nature are presented. The topics discussed are rather simple problems, but it is hoped that the present interdisciplinary approach attracts a wide audience.

I am indebted to many people in my studies of the subjects discussed in this monograph and in preparing this monograph. I would like to express my gratitude to Prof. Akimichi Takemura, the editor of this volume, for his continuous encouragement throughout the preparation of this work. Professor Akinobu Shimizu and Prof. Koji Tsukuda read a draft of this work and gave careful and useful comments. I would like to thank Mr. Yutaka Hirachi of Springer Japan for his help and patience. This work was supported in part by Grants-in-Aid for Scientific Research 15K05013 from Japan Society for the Promotion of Science.

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