Tobias Hanning

High Precision Camera Calibration

VIEWEG+TEUBNER RESEARCH

Tobias Hanning

High Precision Camera Calibration

VIEWEG+TEUBNER RESEARCH

Bibliographic information published by the Deutsche Nationalbibliothek
The Deutsche Nationalbibliothek lists this publication in the Deutsche Nationalbibliografie;
detailed bibliographic data are available in the Internet at <http://dnb.d-nb.de>.

Habilitation thesis University of Passau, 2009

1st Edition 2011

All rights reserved

© Vieweg+Teubner Verlag | Springer Fachmedien Wiesbaden GmbH 2011

Editorial Office: Ute Wrasmann | Sabine Schöller

Vieweg+Teubner Verlag is a brand of Springer Fachmedien.

Springer Fachmedien is part of Springer Science+Business Media.

www.viewegteubner.de

No part of this publication may be reproduced, stored in a retrieval system or transmitted, in any form or by any means, electronic, mechanical, photocopying, recording, or otherwise, without the prior written permission of the copyright holder.

Registered and/or industrial names, trade names, trade descriptions etc. cited in this publication are part of the law for trade-mark protection and may not be used free in any form or by any means even if this is not specifically marked.

Cover design: KünkelLopka Medienentwicklung, Heidelberg

Printing company: STRAUSS GMBH, Mörlenbach

Printed on acid-free paper

Printed in Germany

ISBN 978-3-8348-1413-5

Contents

List of symbols	xi
1 Introduction	1
1.1 Motivation	1
1.2 Outline	2
1.3 Contribution	4
2 Modelling the camera mapping	5
2.1 Geometric optics for computer vision	5
2.1.1 The “thin lens” assumption and first order optics	5
2.1.2 The circle of confusion	8
2.1.3 Image acquisition	9
2.1.3.1 The sensor array	9
2.1.3.2 A simplified sensor model	11
2.1.3.3 The sensor array as coordinate system	12
2.2 The pinhole camera model	13
2.3 Third order optics and thick lenses	15
2.4 The pinhole camera model with distortion	17
2.4.1 Definition	17
2.4.2 Radial distortion	18
2.4.3 Radius transformations	19
2.4.4 Other distortion functions	21
2.4.4.1 Misaligned thin lens	21
2.4.4.2 Misaligned lens systems	22
2.5 Inverting the camera mapping	23
2.6 The pinhole camera model in homogeneous coordinates	25

3	Error functions for camera calibration and 3D reconstruction	27
3.1	Introduction	27
3.2	Projective and re-projective error	27
3.3	Euclidean error	29
3.4	Error functions for camera calibration and 3D-reconstruction . . .	34
3.4.1	Calibration error functions	34
3.4.2	Reconstruction error functions	37
3.5	Non-linear optimization	38
4	Initial values for camera calibration problems	40
4.1	Introduction	40
4.2	The two stage method of Tsai	42
4.3	An initial image transformation by direct linear transformation . .	47
4.4	An initial image transformation from homographies	50
4.4.1	Introduction	50
4.4.2	Two necessary conditions for planar targets	50
4.4.3	Zhang's initial value	52
4.4.4	An initial image transformation with known center and zero skew	54
4.4.5	An initial image transformation with known aspect ratio and no skew	55
4.4.6	An initial image transformation with known aspect ratio and unknown skew	56
4.4.7	An initial image transformation with no skew	58
4.4.7.1	A straight forward constraint	58
4.4.7.2	A solution by a linear least squares problem with Cholesky decomposition	59
4.4.8	Experimental results	60
4.4.8.1	Overview	60
4.4.8.2	Simulations	61
4.5	An initial value for the extrinsic camera parameters	64
4.5.1	Introduction and problem statement	64
4.5.2	Standard pose estimation	64
4.5.3	An algebraic re-projective approach for regular grids . . .	65
4.5.4	An optimal solution w.r.t. Euclidean error for 1D targets .	70
4.6	An initial solution for the distortion	72
4.6.1	Introduction	72
4.6.2	Zhang's initial solution for the radial distortion	72

4.6.3	An optimal initial solution for all distortion parameters . . .	74
4.7	Camera calibration with distortion as a semi-linear problem	77
4.7.1	Parameter reduction by semi-linear optimization	77
4.7.2	Experimental results	78
4.7.2.1	Results for the normal setup	79
4.7.2.2	Results for the webcam setup	85
4.7.2.3	Results for the wide angle setup	89
5	Calibration of a stereo camera system	91
5.1	Introduction	91
5.2	Epipolar geometry	91
5.3	Epipolar Curves	95
5.4	Stereo camera calibration with multiple targets	97
5.5	Extrinsic stereo camera calibration with generalized epipolar constraints	98
5.5.1	A two step algorithm	98
5.5.2	A one step algorithm	100
5.5.3	Application and results	101
5.6	Extrinsic stereo camera calibration with respect to the projective error	103
5.7	Extrinsic and intrinsic stereo camera calibration	105
6	Non-standard camera models	107
6.1	Introduction	107
6.2	Feature point extraction	110
6.2.1	Standard feature point extraction	110
6.2.2	Model based extraction of isolated squares	113
6.2.3	Appropriability of the feature point extraction methods . .	117
6.2.3.1	Appropriability with respect to the sensor model	117
6.2.3.2	Appropriability with respect to the camera model	117
6.3	The residual distortion	119
6.3.1	The point spread function by first order optics	119
6.3.2	Other sources of residual distortion	126
6.3.3	Experimental results	126
6.4	Spline correction	134
6.4.1	Motivation and related work	134
6.4.2	A depth-dependent distortion term	134

6.4.3	Depth-dependent distortion correction for the projective and re-projective error function	135
6.4.4	The tensor spline space	135
6.4.5	Tensor splines for the re-projective depth-dependent distortion	137
6.4.6	Spline correction for the Euclidean error	138
6.4.7	The viewing ray for spline corrected cameras	139
6.4.8	Spline correction for stereo reconstruction	139
6.4.9	Disadvantages of the spline correction	140
6.5	A two-plane distortion model	144
6.5.1	Motivation and related work	144
6.5.2	The plane $\{z = -1\}$	145
6.5.3	Distortion mappings in $\{z = 1\}$ and $\{z = -1\}$	146
6.5.4	The re-projection w.r.t. the two-plane distortion	147
6.5.5	Error functions for the two-plane distortion model	148
6.5.5.1	The projective error	149
6.5.5.2	The Euclidean error	149
6.5.5.3	The projected Euclidean error	150
6.5.5.4	The normalized Euclidean error	150
6.5.5.5	Depth-dependence of the two-plane distortion model	153
6.5.6	Calibration algorithm	155
6.6	A generic multi-plane camera	156
6.6.1	Introduction and related work	156
6.6.2	From the image to a reference coordinate system	156
6.6.3	Tensor spline approximation of the coordinate transformation	159
6.6.4	A calibration setup for the generic multi-plane camera	159
6.7	Experimental results	161
6.7.1	Setup	161
6.7.1.1	Calibration setup for the standard camera model	161
6.7.1.2	Calibration setup for the spline correction	161
6.7.1.3	Calibration setup for the two-plane distortion model	161
6.7.2	Results for spline corrected cameras	162
6.7.2.1	Prototype reconstruction	162
6.7.2.1.1	In-plane spline correction	162
6.7.2.1.2	3d spline correction	164

6.7.2.2	Stereo reconstruction	171
6.7.3	Results for the two-plane distortion model	173
6.7.3.1	Stereo reconstruction	173
6.7.3.2	Point to point error	173
6.7.3.3	Angles of reconstructed planes	177
6.7.3.4	Other test series	180
6.7.3.5	Planarity test	187
6.7.3.6	Prototype reconstruction	192
7	Conclusions	197

Abstract

The main purpose of this work is to determine the camera mapping for optical measurement objectives. The standard approach models the camera mapping as a pinhole camera with distortion. We formulate different error functions for the pinhole camera model. Minimizing all error functions introduces a non-linear optimization. Therefore, we present initial values for the intrinsic and extrinsic camera parameters including distortion. In particular, the distortion can be determined by a linear least squares problem. This yields a semi-linear approach to camera calibration.

Stereo camera calibration introduces an additional constraint, which is used as epipolar line constraint in the literature. We extend this constraint to epipolar curves and present some calibration approaches for a stereo camera setup. These include the epipolar curve constraint.

When modelling the camera as a pinhole with distortion, we observe a residual error. We show that this error depends on the depth of the observed object. Thus, we present two approaches to introduce a depth-dependent distortion model: First, we propose a spline correction of the residual error, second, we suggest a two-plane distortion model. Several experimental results support both approaches.

Symbols

$A_{i,j}$	the element of the i -th row and j -th column of a matrix $A \in \mathbb{R}^{n \times m}$
$\operatorname{argmin}_{x \in X} g(x)$	$:= \{x_0 \in X : g(x_0) = \min \{g(x) \mid x \in X\}\}$
$\mathcal{C}_\infty(E, F)$	the set of functions from E to F which are differentiable for all degrees of differentiation
CCS	camera coordinate system
$\operatorname{dist}_d(A, B)$	$:= \inf \{\ a - b\ \mid a \in A, b \in B\}$ for $A, B \subset \mathbb{R}^d$
ICS	image coordinate system
$\mathcal{K}$	the set of all pinhole cameras
$\mathcal{K}_\Delta$	the set of all pinhole cameras with distortion model Δ
$\mathcal{L}^2(E, F)$	the set of all integrable functions $f : E \rightarrow F$ with $\int f ^2 d\lambda < \infty$
$\mathcal{L}(\mathbb{R}^3)$	$:= \{l \subset \mathbb{R}^3 \mid l \text{ aff in } \mathbb{R}^3, \dim(l) = 1\}$ the set of all lines in $\mathbb{R}^3$
$\mathcal{L}(o)$	$:= \{l \in \mathcal{L}(\mathbb{R}^3) \mid o \in l\}$ the set of all lines in $\mathbb{R}^3$ containing the point $o \in \mathbb{R}^3$
$l(p, q)$	$:= \{x \in \mathbb{R}^n \mid \exists \lambda \in \mathbb{R} : x = q + \lambda(q - p)\}$ the line defined by two point $p, q \in \mathbb{R}^n$
$\tilde{p} := (p_x, p_y, 1)^t \in \mathbb{R}^3$	for a point $p = (p_x, p_y) \in \mathbb{R}^2$
$\tilde{p}^{-1} := (p_x, p_y, -1)^t \in \mathbb{R}^3$	for $p = (p_x, p_y) \in \mathbb{R}^2$ (see page 146)
$\mathbf{O}_3$	orthogonal group in $\mathbb{R}^{3 \times 3}$
$\mathbb{P}^n$	the (real) projective space $\mathbb{R}^{n+1} \setminus \{O\} / \sim$ with $p \sim q \Leftrightarrow \exists \lambda \in \mathbb{R} \setminus \{0\} : \lambda p = q$
$\hat{p} = \begin{pmatrix} u \\ v \end{pmatrix}$	with $(u, v, 1)^t = \Pi_z(p)$ for a point $p = (x, y, z) \in \mathbb{R}^2 \times \mathbb{R} \setminus \{0\}$

$\mathcal{P}(\mathbb{R}^2, \mathbb{R}^2)$	the vector space of all polynomials from $\mathbb{R}^2$ to $\mathbb{R}^2$
$\mathbf{P}$	a finite set of points in $\mathbb{R}^3$ determining a calibration pattern (“model”)
$\text{Proj}_L p$	the orthogonal projection of $p \in \mathbb{R}^3$ on the line L
$Q_t := \begin{pmatrix} 0 & -t_3 & t_2 \\ t_3 & 0 & -t_1 \\ -t_2 & t_1 & 0 \end{pmatrix}$	matrix with $Q_t x = t \times x$ for all $x \in \mathbb{R}^3$ with $t = (t_1, t_2, t_3)^t \in \mathbb{R}^3$
$\mathbb{R}_+ := [0, \infty[$	non-negative real numbers
$\mathbb{R}_+^* :=]0, \infty[$	positive real numbers
RCS	reference coordinate system
$\begin{pmatrix} R & t \end{pmatrix} \in R^{3 \times 4}$	matrix where the first three columns equal the columns of $R \in \mathbb{R}^{3 \times 3}$ and the last column equals $t \in \mathbb{R}^3$
SO_3	special orthogonal group in $\mathbb{R}^3$ (rotation group)
$\mathcal{T}(\mathbb{R}^3, \mathbb{R}^3)$	isometric coordinate transformations in $\mathbb{R}^3$
$u \times v$	the cross product of two vectors $u, v \in \mathbb{R}^3$ for $u = (u_x, u_y, u_z)^t = (v_x, v_y, v_z)^t$ it is $u \times v = \begin{pmatrix} u_y v_z - u_z v_y \\ -(u_x v_z - u_z v_x) \\ u_x v_y - u_y v_x \end{pmatrix}$
$\text{span}(v_1, \dots, v_n)$	$\{\lambda_1 v_1 + \dots + \lambda_n v_n \mid \lambda_1, \dots, \lambda_n \in \mathbb{R}\}$ the linear span of $v_1, \dots, v_n$
w.r.t.	with respect to
$\{z = 0\}$	$:= \{(x, y, z) \in \mathbb{R}^3 \mid z = 0\}$
$\{z = 1\}$	$:= \{(x, y, z) \in \mathbb{R}^3 \mid z = 1\}$