A Scenario Tree-Based Decomposition for Solving Multistage Stochastic Programs with Application in Energy Production

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Debora Mahlke

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for Solving Multistage Stochastic Programs

VIEWEG+TEUBNER RESEARCH

Stochastic Programming

Editor:

Prof. Dr. Rüdiger Schultz

Uncertainty is a prevailing issue in a growing number of optimization problems in science, engineering, and economics. Stochastic programming offers a flexible methodology for mathematical optimization problems involving uncertain parameters for which probabilistic information is available. This covers model formulation, model analysis, numerical solution methods, and practical implementations. The series "Stochastic Programming" presents original research from this range of topics.

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Abstract

This thesis is concerned with the development and implementation of an optimization method for the solution of multistage stochastic mixed-integer programs arising in energy production. Motivated by the strong increase in electricity produced from wind energy, we investigate the question of how energy storages may contribute to integrate the strongly fluctuating wind power into the electric power network. In order to study the economics of energy storages, we consider a power generation system which consists of conventional power plants, different types of energy storages, and an off-shore wind park which supplies a region of certain dimension with electrical energy. On this basis, we aim at optimizing the commitment of the facilities over several days minimizing the overall costs.

We formulate the problem as a mixed-integer optimization program concentrating on the combinatorial and stochastic aspects. The nonlinearities arising from partial load efficiencies of the units are approximated by piecewise linear functions. In order to account for the uncertainty regarding the fluctuations of the available wind power and of the prices for electricity purchased on the spot market, we describe the affected data via a scenario tree. Altogether, we obtain a stochastic multistage mixed-integer problem (SMIP) of high complexity whose solution is algorithmically and computationally challenging.

The main focus of this thesis is on the development of a scenario tree-based decomposition approach combined with a branch-and-bound method (*SD-BB*) for the solution of the SMIP described above. This novel method relies on the decomposition of the original formulation into several subproblems based on the splitting of the scenario tree into subtrees. Using a branch-and-bound framework which we extend by Lagrangian relaxation, we can solve the problem to global optimality. In order to support the solution process, we investigate the polyhedral substructure which results from the description of switching processes in a scenario tree formulation yielding

a complete linear description of the polytope. Furthermore, we develop an approximate-and-fix heuristic which generates feasible solutions of the problem with low computational effort. In the sequel, we specify the implementation of the *SD-BB* algorithm where we exploit problem-specific as well as algorithmic properties to make the method successful. Although our algorithm has originally been devised for a specific kind of problem, the general framework can be applied for the solution of a wide range of related problems. Finally, we evaluate the performance of the developed methods based on a set of test instances close to reality. Applying the *SD-BB* algorithm, we consider instances with a time horizon ranging from six hours up to several days using a time discretization of 15 minutes.

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