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Geometry and Dynamics of Groups and Spaces

In Memory of Alexander Reznikov

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Preface

This book is a collection of selected papers on recent trends in the study of various branches of mathematics. Most of the authors who contributed to this volume were invited speakers at the International Conference "*Geometry and Dynamics of Groups and Spaces. In Memory of Alexander Reznikov*" which was held at the Max Planck Institute for Mathematics (Bonn), Germany, in September 22–29, 2006.

Alexander (Sasha) Reznikov (1960–2003) was a brilliant mathematician who died unfortunately very early. This conference in his remembrance focused on topics Sasha made a contribution to. In particular: hyperbolic, differential and complex geometry; geometric group theory; three dimensional topology; dynamical systems.

The list of participants: Belolipetsky Mikhail; Bismut Jean-Michel; Boileau Michel; Breuillard Emmanuel; Danilenko Alexandre; Delzant Thomas; Deninger Christopher; Esnault Hélène; Franks John; Gal S.R.; Gunesch Roland; Hai Phung Ho; Kaimanovich Vadim; Kapovich Michael; Klingenberg Wilhelm; Makar-Limanov Leonid; Milman Vitali; Morishita M.; Moree Pieter; Navas Andres; Neretin Yuri; Papazoglu Panagiotis; Parker John R.; Porti Joan; Rosellen Markus; Simpson Carlos; Swenson Eric L.; Szczepanski A.; Tomanov Georges; Tsygan Boris; Verjovsky Sola Alberto; Wang Shicheng; Zakrzweski Wojtek.

Talks took place in an informal and constructive atmosphere, and it was a pleasure to see discussions taking place between groups of participants all over the Institute and at all times of day. The exceptional quality of the lectures and the great interest they generated among the conference participants gave us, the organizers of the conference, the idea of this book. The editors wish to record their thanks to the staff at the Max-Planck Institute and to the many researchers who participated for all their efforts in making this such a stimulating experience.

This volume is a collection of papers which aims at reflecting the present state of the art in a most active area of research at the intersection of several branches of mathematics. In the volume we include an unpublished manuscript "Analytic Topology of Groups, Actions, Strings and Varietes" by Sasha (this article is posted, essentially, in the form it appeared as the ArXive preprint math.DG/0001135 in January of 2000; the referee corrected obvious typos, updated and added few references, and made several comments in the form of footnotes and italicized remarks) and some short speeches/recollections about Sasha.

Topics discussed in the book include analytic topology of groups, actions, strings and varieties, category theory, homological algebra, the quantum dilogarithm, Chow groups, parabolic bundles, quantum spaces over non-archimedean fields, Nori's fundamental group scheme, Baumslag-Solitar groups, finitely generated branch groups, Serre's property (FA), hypoelliptic equations, Hodge theory, index theory and related fixed point theorems, determinants and determinant bundles, analytic torsion, Kleinian groups, hyperbolic manifolds, geometric functional analysis, Chern character and cyclic homology, Milnor invariants and l-class groups, geodesic flow, operads, algebras, inner cohomomorphisms, symmetry and deformations in noncommutative geometry, Chebyshev-Dickson polynomials, convolution operators, lattices, discrete harmonic functions, ergodic transformations and rank-one actions, hyperbolic surfaces, eigenfunctions, non-Archimedean metric spaces.

The articles collected in this volume should be of interest to specialists in such areas of mathematics as algebra, dynamical systems, geometry, group theory, functional analysis, number theory, probability theory and topology. The broad spectrum of topics covered should also present an exciting opportunity for graduate students and young researchers working in any of these areas who are willing to put their research in a wider mathematical perspective.

Alexander (Sasha) Reznikov (1960–2003)

Alexander (Sasha) Reznikov was born in Kiev (Ukraine, former USSR) on January 14, 1960. From a very early age he was fascinated with mathematics. It took him only 8 years (instead of the usual 10 years) to finish the primary and secondary schools. In 1975 (the last year of the secondary school), Sasha won the second prize at the International Mathematical Olympiad. Because of this, according to the former Soviet rules he got the right to be admitted as a student to any University of the Soviet Union without the entrance exams. For this reason, he, luckily, was able to avoid the enormous obstacles faced by other Jewish students who tried to enter prestigious Soviet universities during the 1970s and 1980s.[1] Sasha was admitted to Kiev State University at the age of 15 and was a brilliant student during his years at the University. He successfully participated in the student mathematical life and started to do research early on. Despite his success, the fact that he was a Jew effectively barred him from graduate programs and jobs at research institutes. Sasha got a job at a state planning institution which had nothing to do with mathematics. Outside of his working hours there, he visited the Kiev Institute of Mathematics and worked on his PhD thesis under the supervision of Myroslav Gorbachuk. Around that time he became interested in Jewish history and joined a small group studying the history of the state of Israel. Very soon, the Soviet secret service discovered this activity and reported Sasha to his employer. He was forced to quit his job and to leave Kiev. Sasha started to travel the country. He worked in Lithuania, Tajikistan and other remote regions, doing mostly manual work, unable to continue his research in mathematics. Fortunately, the times were changing and starting from 1988 Soviet Jews were again allowed to emigrate. Sasha emigrated to Israel in 1989 and already a year later completed his PhD thesis at the Tel Aviv University under the supervision of Vitali Milman. After spending a year as a postdoc at ICTP in Trieste, he became a lecturer at the Hebrew University in Jerusalem. He remained there until he took a chair in Durham in 1997. Sasha Reznikov, Professor of Pure Mathematics at the University of Durham, died on 5 September 2003, at the age of 43.

We will now describe briefly the most important mathematical contributions of Sasha Reznikov. He started as a classical Riemannian geometer and one of his most impressive results is a proof of the so-called weak Blaschke conjecture [Re19]. A compact Riemannian manifold is called a *Blaschke manifold* if the length of the maximal geodesic segment α starting at any point p, is independent of p and α. The problem is whether the only possible Blaschke manifolds are spheres and projective spaces over the reals, complex numbers, quaternions and Cayley numbers, equipped with their canonical metrics. Sasha proved that all Blaschke manifolds have the same volume as the spheres or projective spaces on which they are modelled.

Sasha Reznikov's most influential work is his proof of Spencer Bloch's conjecture on representations of the fundamental group of an algebraic variety. The proof

[1] This situation is described in the book by M. Shifman "You Failed Your Math Test, Comrade Einstein: Adventures and Misadventures of Young Mathematicians".

is a remarkable combination of arithmetic and analytic methods. Sasha proved that for any smooth complex projective variety, and any representation of its fundamental group into $\mathrm{SL}(2, \mathbf{C})$, the second Chern class in the Deligne cohomology of the associated holomorphic bundle is torsion [Re14]. More generally, he later showed that for all flat bundles on a smooth projective variety, all Chern classes in the Deligne cohomology, except the first one, are torsion [Re17]. These are outstanding results which opened new directions of the research in this field. C. Soulé gave a talk at the Bourbaki Séminaire devoted to this theorem of Reznikov, [So].

For these results Sasha Reznikov was awarded a sectional talk at the European Congress of Mathematics in Barcelona in 2000 [Re2].

In the middle of 1990s he became interested in the geometry and topology of 3-manifolds and geometric group theory. He attempted to prove the famous Haken–Waldhausen–Thurston conjecture that any irreducible 3-manifold with infinite fundamental group has a finite covering with positive first Betti number (such 3-manifolds are called *virtually Haken*)[2]. During these years, he wrote a series of important papers [Re4], [Re5], [Re6], [Re7] and [Re9]. In [Re9] he proved several restrictions on manifolds which are not virtually Haken. In particular, it follows from Reznikov's theorem that if the manifold M is hyperbolic and is not virtually Haken, then for every prime number p there exists a finite covering $N \to M$ such that rank $H_1(N, \mathbb{F}_p) \geq 4$. In the paper [Re4] he discussed similarities between the 3-manifold topology and the theory of number fields. These are illustrated by several interesting examples concerning Heegaard splittings of 3-manifolds. In [Re8] Sasha developed an analogy between the symplectomorphism groups and linear groups; he proved in particular that the inclusion of the compact Lie group $PSU(n+1)$ into the symplectomorphism group of the complex projective n-space is injective on the rational homology.

The final work of Sasha Reznikov "Analytic topology of groups, actions, strings and varieties" was written in 2000 and remained unpublished since then. It contains many important ideas and results, in particular it was instrumental in study of the property T (Kazhdan property) of groups of diffeomorphisms of the circle $\mathbb{S}^1$. We are very glad that this paper is included in our Proceedings and we thank the referee for his careful reading of the paper and many useful remarks and corrections. We keep the referee's comments in the form of footnotes and italicized remarks.

Sasha Reznikov wrote 34 mathematical papers. Most of them are written in a very short period during the 1990s. Their mathematical scope is enormous; the results of Sasha Reznikov belong to several different areas of mathematics: Riemannian and symplectic geometry, 3-dimensional topology, geometric group theory, algebraic geometry and dynamical systems. This diversity of Sasha's research interests is reflected by the variety of topics covered by the articles which appear in these Proceedings of the Conference dedicated to the memory of Sasha Reznikov. The Conference was held at the Max-Planck Institute of Mathematics

[2]This problem remains open.

in Bonn in 2006 from September 22 until September 29 as a satellite conference to the annual meeting of the German Mathematical Society which in 2006 took place in the period September 18–September 22 at the University of Bonn.

We are deeply thankful to Sasha's mother, Ida Reznikova, for providing us with the details of his biography which we used in the introduction of the volume.

Many thanks are due to all contributors, especially to Michael Kapovich and Boris Tsygan for their contributions and help with the preparation of the volume. We are very grateful to Birkhäuser for publishing this volume in the "Progress in Mathematics" series.

Mikhail Kapranov, Yale
Sergiy Kolyada, Kiev
Yuri Manin, Bonn/
Northwestern
Pieter Moree, Bonn
Leonid Potyagailo, Lille

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Short Recollections about Sasha

Ida Reznikova – (Sasha's mother)

Sasha was born in Kiev on 14 January 1960. He was a quiet child, and seemed similar in his development to our other children. He was bored in elementary school, and the teacher advised us to let him skip a grade. In the sixth grade he again became bored, and the mathematics teacher insisted that he skips grade again, so he went to the eight grade of the mathematical school N 145.

He was so fascinated with mathematics that he was invited to the International Mathematical Olympiad in the last, 10th grade. The Soviet team shared with the US one the third and fourth prizes, while Sasha personally was awarded the second prize. He wanted to enter the Moscow University. He had the right to be admitted, as the winner of the Olympiad. But since he was only 15 when he graduated from the high school, he went to the Kiev University, as we lived in Kiev.

After attending all the lectures once, Sasha realized that he has nothing to gain by going to the lectures, as he knew the material. He regretted very much that he did not enroll in Moscow University, which he could have done without entrance exams but missed the chance. Sasha remained in Kiev.

After graduating summa cum laude, he could not find a job in any research institution, although he was a star at the university. The Dean told us right away that "people such as Sasha" (i.e., Jews) will not be admitted to any research institution, so Sasha got a job in a state planning establishment which had nothing to do with mathematics.

... After a while he met a woman with background in humanities. He learned that she wants to emigrate to Israel, but he had no plans to emigrate himself, and neither did anyone in our family at that time. What he did not know was that she was followed by the KGB, because she insisted on her right to emigrate.

The KGB reported Sasha to his employer, that he keeps company of "undesirable" people. They wanted to use Sasha to uncover and punish all the people involved, so they put pressure on the administration of his institute. The administration organized a staff meeting with public criticism of Sasha's lifestyle. Among other things, he was accused of wearing jeans all the time. Jeans were considered an expensive bourgeois excess, it is hard to believe this now, but it was for real. Sasha's father was a retired Army officer (32 years of service), he had a sizable military pension while still working in his retirement, and I worked as well. So we could afford to buy our children expensive items. Anyway, Sasha was expelled from the Komsomol (Young Communist League). The local newspaper published a very negative article about Sasha. The situation at work became intolerable and he wanted to leave his job. They summoned his father, put him to shame for raising "such a son". All the service that Sasha did to the university, his excellence at the olympiads were ignored. The year before he was admitted to the Institute of Mathematics for graduate study, and the people there treated him well and wanted him to continue his studies. They were decent people who understood that he was picked at for no reason.

In August 1985 Sasha packed his things and left, without telling us where. He wrote me a letter with no return address, saying that he leaves, that he can stand up for himself and support himself if necessary.

His absence dragged on, we started a search, and when we located him, Misha, his brother, came to see him with hope of influencing him somehow. But Sasha did not return home, he worked at a meteorological station for a pittance. He was afraid to look for a better job knowing that his expulsion from the Komsomol would come out. He found a place in a small town in Lithuania. Misha said that Sasha looks OK, he is fine, does not want to return and does not want any visitors.

In the Fall of 1986 I decided to come see Sasha, although I did not know the address. Misha was strongly opposed to the idea. I knew only that Sasha lives in Turmantas (Lithuania). Not knowing how would I find him there, I did not take many things with me, and also did not take a lot of money or food. But Turmantas was a small village and I easily found Sasha there. He looked good, one could tell he spent a lot of time outdoors, I was glad to see that. But the hut where he lived, was truly horrible, it had an outhouse in the backyard. His trousers were ragged, I was walking around the hut thinking what to bring next time.

He received me well, told me about his life. I was in a good mood seeing my son so tall and handsome, with bright shining eyes.

In a couple of days, Borya (our eldest son) took a couple of days off work and went to see him with a full suitcase of necessities and a considerable sum of money, but he did not find Sasha. Borya was told that Sasha quit his job and left for an unknown destination. We could not imagine where could he be and how to find him. Someone suggested to contact the police, which I was reluctant to do so as not to get Sasha in more trouble. But there was no other option.

The police helped us, and in 1987 I learned that Sasha was in Dushanbe (Tajikistan). This was in summer. Misha who flew in to see him, had to keep all money and things and fly back. After this, Sasha went into hiding again. I did not believe that and flew to Dushanbe the next week with the hope to find him there, but he left already. I spoke with his former coworkers, they spoke with high regards of him, regretted that he left. In fact, he was offered graduate study in the area of agriculture which was related to his job. Whatever he started, he succeeded. Everyone was so fond of him, and I cried when I saw his apartment.

We decided not to bother him for a while, but I could not stand being in the dark for long. I started searching for him again and learned that he is in Tselinograd (Kazakhstan), has a decent job, although still far from his beloved mathematics. I flew to see him some time in 1988. I realized that his only choice for normal life is to leave the USSR where he was forever branded as a “renegade”.

At that time several people began to emigrate, using invitations from Israel. Sasha went to Israel in 1989, as he felt that a mathematician should do creative work while he is still young, without wasting any time. This was the beginning of a very happy period of our life. Sasha resumed contact with us, was calling home. I decided that it would be better if we went to Israel as well since Sasha is there,

wants us to be in his life, this was pure happiness. We emigrated with my husband to Israel in December 1990.

Sergiy Kolyada

For the first time I met Sasha in 1974. I remember that day. We both took part in one of the mathematical competitions ("mathematical battles") which were at that time so popular among high school and university students in Kiev as well as in many other cities of the former Soviet Union.

In this picture you can see Sasha who congratulates Kostya Rybasov, the captain of our winning team of Ukrainian physical-mathematical boarding high school at the Kiev State University. Here, close to Sasha, you can see Borya Tsygan. So, from that time we were close friends with Sasha.

By the way, in those years at the Ukrainian high schools there were many students devoted to mathematics. Later many of them became outstanding mathematicians. Apart from Borya Tsygan and Kostya Rybasov already mentioned let me add at least the names of Sasha Blokh and Mikhail Lyubich from those, who were of the same age as we. Sasha Goncharov, another friend of Sasha Reznikov, was one year younger.

After we finished our high school studies in 1975, Sasha, Borya Tsygan, Kostya Rybasov, me and some others became students of the Kiev State University, while Lyubich and Blokh chose the Kharkov State University.

I will not speak on the life of Sasha during the Soviet era. Let me only add that, for several reasons, at that time it was not easy for students of the Kiev State University to work seriously in mathematics, although there were of course many good mathematicians in Kiev. Being students, we were trying to participate in good scientific seminars, for instance in the seminar of Sergei Samborski at the Kiev Polytechnic Institute. I think we were second year students when Sasha wrote his first paper (on inequalities in triangles). I remember it very well, since

Sasha asked me to read his paper and tell him my opinion before submitting it to the journal "In the world of Mathematics". Though Sasha was in fact two years younger than other students in our group, he always was the life and soul of the group. He was an extraordinary person.

After graduation from the university, Sasha was a PhD student at the Institute of Mathematics of the Ukrainian Academy of Sciences (a correspondence post-graduate study). Formally, his supervisor was Prof. Myroslav Gorbachuk, but he also had very close scientific contacts with Prof. Georgii Kats, Prof. Aleksei Pogorelov and his group in Kharkov. Sasha often used to give lectures at scientific seminars in Kharkov.

Unfortunately, that time a tragic event entered his life and this badly influenced all the rest of his life. Since in those years I was not in a contact with him, I learnt about this matter too late... This is briefly described in his mother's recollection above.

Having left Kiev, Sasha interrupted contacts with all of us. The reason was that he was afraid that we could have problems with the KGB because of him. In 1991 he sent me a letter from Trieste. To illustrate the situation, let me say that the letter was not sent by post but was brought me by a Chinese mathematician and the letter was put into three (!) envelopes.

From that time on, until his death we had been exchanging letters with Sasha regularly. Unfortunately, since 1985 I could not meet him personally any more, but I heard Sasha was very active, making a name for himself in mathematics, talking to many mathematicians.

I believe that the conference and the volume in memory of Sasha Reznikov will significantly contribute to the development of Mathematics.

Boris Tsygan

I knew Sasha since sixth or seven grade. We went to different schools and lived in different parts of Kiev but we met at mathematical competitions. We went to two Ukrainian Olympiads together as members of the Kiev team. I remember that the mathematical battle that is described in this preface ended for us in a police station. We were detained for playing cards in one of the parks for which the city was famous. That particular park, or rather a forest resembling Bois de Boulogne, was on the grounds of the Exhibition of Advanced Experience. I cannot recall the policies or the logic behind the detention, but it seems that minors playing cards were frowned upon. To recall a touching sign of the times, our cards were made of perfocards that we used in our computer science program at school. In 1975 we entered the same class of the Kiev University. That Sasha did not go to Moscow was a disappointment for him but a blessing for me.

Sasha was my first mentor in advanced mathematics but he was to me much more. He was the first real mathematical talent that I met. More generally, he was the first person in my life with a talent to create intangible beautiful things from the inside of his head. To meet such a person early in one's life one has to be lucky, especially if one lives outside of a few great cities. I was duly impressed.

Sasha was always aiming high, both mathematically and otherwise. Back then, when I read biographies of famous mathematicians, I noticed qualities and traits that I recognized in Sasha (I still do). I also remember thinking that perhaps the most interesting thing I will ever write will be memoirs about him.

We had a group of several undergraduates studying math. Sasha was an undisputed leader. He would go to Moscow, then come back and tell us what he learned there from discussions with prominent mathematicians, in particular with Joseph Bernstein. We were also reading and discussing books. Our studies tended towards representation theory and geometry of manifolds, both ignored in the undergraduate University curriculum at Kiev at the time. When we started the second or third year, S.N. Samborsky and Yu.L. Daletsky at the Polytechnical Institute organized a learning seminar on geometry and topology of manifolds. We started by studying the book Analysis on Complex Manifolds, by Wells, and Langs $SL_2(\mathbf{R})$. I was entranced (for Sasha, I think, all this was interesting but not that new). Later we attended special topics courses and seminars by Yu.A. Drozd at the University and A.N. Tolpygo at the Polytechnical, or later at A.N.'s place. The topics were algebraic geometry, algebraic groups, Lie groups, etc. Looking back, I can compare these to graduate courses and graduate seminars at the best universities in the world. This was due partly to brilliant teaching and partly to Sasha's stellar presence. After the meetings we had long walks discussing poetry, history, and mathematics.

It was with Sasha that I went to my first conference, or rather to a Summer school in Kazan, Tatarstan. To make it in time, one of Sasha's exams had to be taken two days earlier. This was theoretical mechanics. Yu.L. Daletsky noted that the professor was his classmate, and phoned her from his office. She was friendly and nice and told us to come to take the exam early. When we came and the exam started, suddenly we were subjected to a frontal assault that I had experienced only once before or since, at my entrance exams at Moscow. After a brutal examination, we are told that we failed but we are more than welcome to come again when we learn the subject. Those long June days in Kiev, when we were sitting in Shevchenko Park opposite the University and discussing classical mechanics with Sasha, are among my best memories. We tried again, and failed. (Sasha: "But Arnold writes..." Professor: "Go to Arnold and ask him to pass you"). Then came the day of my (not Sasha's) regular exam. I come, enter the classroom, get under yet another frontal assault, and exit with yet another failing grade. Everybody is shocked, both me and my classmates who have never seen me fail a test. One of them can't believe that this happened, and asks me to swear by crossing myself. Dazed and confused, I oblige her, and get a strong public rebuke from Sasha: how can you, a Jew... One has to understand the atmosphere back then, in 1977, to appreciate the audacity of this (not that the sign of the Cross was wildly popular with the authorities). Two days later we come to the exam with Sasha's group, get a friendly and charming treatment and good grades, and leave for Kazan two days after the Summer school started.

When it was the time to graduate, we were told by head of the chair of Analysis that he called all the places that asked for "young specialists" and only one was willing to take a Jew. All other options were at high schools. He was sorry and really did not know what to do. This left Sasha and me in a difficult situation. Working at a place that had nothing to do with mathematics was a common thing for many, especially for Jews; arrangements would be made for the person to have enough time for research. The teaching load at high school did not allow this. I recall that there was no shadow of ill will between us. The situation was resolved when we arranged for me to work at the computing center of the factory where my father worked. My position was proudly called mathematician, the salary was slightly better than average, the supervisor was a PhD in functional analysis. I did have to show up and to work there but I got some time for research and was quite happy.

Sasha went to the place for which we were supposed to compete. Two years later I entered the graduate school at Moscow University. Sasha's life took a difficult turn. We did meet from time to time when he was still in Kiev. When he left, we lost contact for years.

We resumed contact when he was in Israel and I at Penn State. In the Spring of 1994 I visited him in Jerusalem. Apart from always exciting mathematical discussions, we talked about many other things, like in our youth. This was another first for me: for the first time in my life I had a conversation with an Israeli. I remember a calm, informed, nuanced, and intelligent view of the country's situation, a sort of talk that I heard sometimes since then from Israelis and not so often from others.

Mathematical conversations with Sasha were always a joy to me. For full disclosure, I do remember a conversation in which mathematical arguments were mixed with (my) threats of violence, but we were fourteen or fifteen at the time. In more mature age, there could be an abrasive word now and then, but my general recollection is of high intellectual charge, generosity, and a sense of high purpose. A compliment from him was not frequent and meant a lot. Often he would listen to your question or your plan of action, think hard, make a few interesting remarks, and then quietly ask: but, in reality, does anyone care? This would help me concentrate my mind on more important things.

Yan Soibelman

My recollections about Sasha are sparse. I recall meeting him for the first time in Kiev, around 1980, at the seminar of Yuri Daletsky. At that time I was a graduate student at the University of Rostov and returned to my relatives in Kiev for vacations (perhaps, it was winter). Sasha gave an "educational" talk about G-structures. I recall his attempt to visualize the latter notion on the blackboard, and then asking the audience: "What is it?". The suggested (by him) answer was "a military unit". "But in fact", said Sasha, "it is a G-structure".

At that time I was interested in papers by Krichever, Novikov and others on non-linear integrable systems. I was surprised to see how enthusiastically Sasha

responded to my questions, since, in my opinion, nobody in Kiev was interested in such things. Later I realized that it was his typical reaction to any new and interesting mathematics.

I found him a very pleasant person, but he explained me that it was all due to his interest in the famous book of Dale Carnegie. "A few years ago I was a completely different person", he told me.

We had more time for discussions four (?) years later, when I returned to Kiev as a fresh PhD, and quickly realized that my Jewish origin made it impossible to find a job there. Sasha was not much lucky for the same reason. He got an obscure job in Kiev. I recall visiting him there once. It was a typical agency, with a lot of people in one room, a lot of noise, and an almost visible atmosphere of "killing" the working time. Sasha's work there had hardly any relation to serious mathematics. Nevertheless he was still very optimistic about his future.

Perhaps during that period (it lasted 1.5 year) we went together to Vinnica, the town of my childhood. In fact we joined a group of teachers from the school, where my mother taught Russian literature. I still have a few photos from that trip. I recall Sasha's attempt to find a synagogue in Vinnica. Since nobody knew where it was, he started to ask people in the street. Only a person who lived in USSR at the beginning of 80's can imagine how pedestrians reacted to that question.

Two years later I returned to Rostov-on-Don, where friends found a job for me at the university. Very soon I heard about Sasha's problems with the KGB in Kiev. He was looking for a place to escape from Kiev. I spoke to my former scientific advisor, Igor Simonenko (without telling him all the details which I did not know myself at the time), and he agreed to take Sasha as a graduate student. Perhaps, things got worse in Kiev, since very soon Sasha asked me about a "not-so-visible" position, e.g., a meteorologist at a station. Me and my wife Tanya tried to find such a position at her agricultural institute in the Rostov region, but at the time when we succeeded, Sasha left Kiev and went into hiding. First time I was sending him letters to Latvia, but then he changed the place, and I lost him for several years until we met in Bonn in 1993.

Jean-Michel Bismut

Je crois bien que c'est lors de l'une de mes premières rencontres avec Sasha qu'au hasard d'une ballade dans le Quartier Latin, nous sommes allés voir un film de Jean Renoir, 'Le déjeûner sur l'herbe'. Je l'avais certes mis en garde sur le fait que sans le secours de sous-titres, il risquait de s'ennuyer. J'ai le souvenir que dans les éclats de rire des spectateurs qui ponctuaient la projection d'une œuvre poétique et grinçante, se détachait, clair, le rire de Sasha. A la sortie, alors que je ne cachais pas ma perplexité, il devait me dire qu'il avait apprécié chaque minute de ce film.

Venu un soir chez nous, alors qu'il était à peine entré et qu'il avait fait connaissance de l'un de mes fils âgé de six ou huit ans, je ne sais plus, nous entendîmes le bruit d'une cavalcade effrénée. Avec mon fils, et sans le secours des mots, il avait improvisé un jeu de poursuite et de guerre, où mon fils avait enfin trouvé à qui parler. Ayant souvent vu passer des mathématiciens, et croyant savoir à qui ils avaient à faire d'ordinaire, mes enfants devaient souvent évoquer entre eux le passage singulier de Sasha Reznikov.

Nous étions allés au Théâtre du Châtelet, à une représentation de 'Fidelio' par le Staatsoper de Berlin, dans une mise en scène qui devait, pour une fois, être à la hauteur des espérances qu'on pouvait y mettre. Au dernier acte, dans le balcon où nous nous trouvions, les portes s'étaient brusquement ouvertes, et nous avions été bousculés par des personnages gigantesques, choristes introduits au dernier moment parmi les spectateurs, entonnant le chœur final. Bouleversés, nous achevâmes la soirée dans un bistro, où j'ai encore le souvenir de nos rires mêlés.

Ce séminaire de Sasha à Orsay me revient aussi en mémoire. Il devait y annoncer la preuve de la conjecture d'hyperbolisation. La veille, tard le soir, il m'avait appelé pour me dire qu'un trou avait été trouvé dans la preuve. Je lui avais proposé d'annuler le séminaire, ce qu'il avait refusé. Crânement, il avait fait face. Ou quelques semaines avant, lorsqu'il expliquait sa preuve, à Jerusalem, dans un contexte d'extrême tension, devant un auditoire souvent perplexe.

Et puis plus rien.

An English translation:
If I remember correctly, it was during one of my first meetings with Sasha that while strolling around the Latin Quarter, we decided to watch a movie by Jean Renoir, 'Le déjeûner sur l'herbe'. I had warned him that without the help of subtitles, he would simply be bored. I vividly remember that among the bursts of laughter coming from the audience watching this poetic and ironic movie, I could also hear the unique laugh of Sasha. At the exit, while I was somewhat perplexed by the movie, Sasha told me that he had enjoyed every minute of it.

One evening he came over to our place. He had just entered the apartment and met one of my sons, who was six or eight years old, I am no longer sure, we suddenly heard the noise of a frantic cavalcade. Without words, he and my son had invented a game of chase and war, in which my son had finally found a suitable partner. Having previously met mathematicians at home, and convinced they knew what to expect, my children would often remember their surprising meeting with Sasha Reznikov.

On another occasion, we had all gone to the Théâtre du Châtelet, to see 'Fidelio' by the Berlin Staatsoper, in a production which, for once, was at the level that one might expect. During the final act, in our balcony, the back doors opened suddenly, and we were pushed aside by gigantic characters, singers who had been introduced into the audience to sing the final choir. Still in shock, we completed the evening in a 'bistro' nearby, and I remember the common bursts of laughter.

The seminar by Sasha at Orsay also comes back to my mind. He was to announce the proof of the hyperbolisation conjecture. The day before, late in the evening, he called to say a gap had been found in the proof. I hinted he could cancel the seminar, which he bluntly refused. Or a few weeks earlier in Jerusalem, when he was explaining the proof to a somewhat skeptical audience, at a time of extreme tension outside.

And then nothing more.

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Sipping Tea with Sasha

Pieter Moree

Dedicated to the memory of Alexander Reznikov

Abstract. In this non-mathematical note I describe how my one joint paper with Sasha arose and how he was the source of inspiration for two other papers.

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1. Summer tea in 1996

One of my favorite moments in my mathematical career occurred when I met Sasha Reznikov for the very first time. It was in the summer of 1996 and at that time the arrival of new visitors at MPIM was not that frequent, so that often I would make a chat with a new visitor. In this case the visitor was Sasha. I asked him what area of mathematics he was working in. He did not like my attempt to pin him down to a particular field of mathematics and told me that he considered himself not to belong to any mathematical field in special, rather he considered himself to be an amateur over a broad range of topics. The subsequent discussion, however, made clear that mathematically I seemed to be very far removed from his mathematical interests. The usual reaction from somebody mathematically quite removed from me on hearing about my number theoretical interests would be disinterest. So I got accustomed to anticipating on this, by just mumbling one sentence and then changing topic. So I mumbled to Sasha that I was working on something called the Artin primitive root conjecture and was about to change topic in the next sentence. However, his reaction could not have been more different than what my prejudices suggested: he was immediately extremely enthousiastic:

'that is PRECISELY what I need !!'

Sasha was close to making a lot of progress on a conjecture of Lubotzky and Shalev. They had made the following conjecture:

Conjecture 1. *Let M be a hyperbolic three-manifold. Let $f(d)$ denote the number of subgroups of index d in $\pi_1(M)$. Then, for all d sufficiently large, $f(d) > \exp(C_1 d)$, where $C_1 > 0$ is a constant that depends at most on M.*

Modulo a number theoretic ingredient, Sasha [8] could prove: suppose the three-manifold M is a rational homology sphere, then there exist $\alpha, C_2 > 0$ such that for infinitely many d we have $f(d) > \exp(Cd^{\alpha})$.

In order to complete his proof he needed a variation of the following celebrated result of Heath-Brown regarding Artin's primitive root conjecture [3].

Theorem 1. *Let q, r and s be three distinct primes. Then at least one of them is a primitive root for infinitely many primes.*

Namely:

Theorem 2. *Let q, r, s be three distinct primes each congruent to* $3 (\mathrm{mod}\ 4)$. *Then for at least one of them, say q, there are infinitely many primes p such that q is a primitive root modulo p and $p \equiv \pm 1 (\mathrm{mod}\ q)$.*

Recall that Artin's primitive conjecture (1927) states that if g is not equal to -1 or a square, then there should be infinitely many primes p such that g is a primitive root modulo p. Thus the above theorems go some way towards establishing this conjecture. (For an elementary introduction to this material, see, e.g., Moree [4] or Ram Murty [6].)

The restriction $3 (\mathrm{mod}\ 4)$ on the three primes is quite essential. If $l \equiv 1 (\mathrm{mod}\ 4)$, then l is not a primitive root modulo p for any prime $p \equiv \pm 1 (\mathrm{mod}\ l)$: by quadratic reciprocity one has $(\frac{p}{l}) = (\frac{l}{p})$ and since $(\frac{p}{l}) = (\frac{\pm 1}{l}) = 1$, it follows that $(\frac{l}{p}) = 1$ and therefore, $l^{(p-1)/2} \equiv 1 (\mathrm{mod}\ p)$, and so l is not a primitive root modulo p.

It was the last condition on p in Theorem 2 that was crucial to Sasha's proof. I quickly realized that the latter result can be obtained by making some minor modifications in Heath-Brown's proof. Two days after our tea discussion I phoned Sasha and informed him that I had solved his problem. His enthusiasm was boundless and he repeatedly told me that I had saved him about a half year of work (since his experience with number theory was not that large, he said he would need a lot of time to study the literature, etc.).

Unfortunately, the original Artin's primitive root conjecture can be presently only proved assuming the Generalized Riemann Hypothesis (GRH) and so I was quite happy that what Sasha needed could be proved unconditionally. A result of the form 'Conjecture 1 is true assuming GRH' would certainly have been a bit comical!

Soon I had written down several pages of material, more or less equalling the number of pages Sasha had written. To me it looked odd to have an appendix about as large as the main text of the paper and so I proposed to Sasha to coauthor the paper, but with the alphabetical order reversed so as to suggest my minor

contribution to the paper. This is what Sasha agreed on and in this way we could write a more homogeneous paper.

Sasha's paper is closely related to his paper [7] where the analogy of covers of three-manifolds and class field theory plays a big role (an analogy that was apparently first noticed by B. Mazur). Sasha and Mikhail Kapranov (at the time also at the institute) were both very interested in this analogy. Eventually, in August 1996, Kapranov and Reznikov both lectured on this (and I explained in about 10 minutes my contribution to Reznikov's proof). I was pleased to learn sometime ago that this lecture series even made it into the literature, see Morishita [5]. Meanwhile several further authors studied the analogy in further detail: Nguyen Quang Do, Sikora and Waldspurger, for example.

After the paper was finished, Sasha submitted it to a top mathematical journal. Much later, the editor informed us that he had not been able to get a referee report and could understand our frustration and would not be insulted if we wanted to publish our paper somewhere else. Ironically, with MathSciNet the paper suffered a similar fate: it does not have a reviewer. (Since H.-W. Henn with whom I discussed Sasha's contribution to the paper told me he was impressed by the range of techniques used by Sasha, I was not completely surprised by this outcome.)

Meanwhile S.-T. Yau, whom Sasha had sent a preprint, had written Sasha that he wanted to publish the paper (provided all the other editors approved it) in the journal he had recently founded: the Asian Journal of Mathematics, This is the reason why the paper eventually ended up in the Asian Journal of Mathematics. The referee of this journal described Sasha's proof as 'ingenious'.

2. Summer tea in 1997

About a year later, in the summer of 1997, I met Sasha again during tea time. Whilst I was happily eating cookies he asked me without much ado whether I knew how the drilling machines of dentists looked like in the old days. (Somehow I immediately lost my appetite for further cookies...) I told him I knew: of lots of arms that can move in all directions. So if one has arms of length $\alpha_1, \ldots, \alpha_m$, then the vertical lengths the dentists arms can make form a subset of the positive values amongst $\pm\alpha_1 \pm \alpha_2 \pm \cdots \pm \alpha_m$. He then told me and Amnon Besser, who also was present, that considerations in work he was doing with Luca Migliorini (then also at the Max-Planck) on the cobordism theory of the moduli space of polygons suggested the following to be true: Let $m \geq 3$. Let $\underline{\alpha} = (\alpha_1, \alpha_2, \ldots, \alpha_m) \in \mathbb{R}^m_{>0}$ and suppose that there is no $\underline{\epsilon} \in \{\pm 1\}^m$ satisfying $\langle \underline{\epsilon}, \underline{\alpha} \rangle = 0$ (inner product is zero). Let $1 \leq i < j \leq m$. Let $\underline{\alpha}_{i,j} \in \mathbb{R}^{m-2}_{>0}$ be the vector obtained from $\underline{\alpha}$ on deleting α_i and α_j. Let

$$S_{i,j}(\underline{\alpha}) := \{\underline{\epsilon} \in \{\pm 1\}^{m-2} : |\alpha_i - \alpha_j| < \langle \underline{\epsilon}, \underline{\alpha}_{i,j} \rangle < \alpha_i + \alpha_j\}.$$

Then Sasha suggested that the cardinality of $S_{i,j}$ should be independent of the choice of i and j. This claim seemed very implausible to Amnon and me and so we immediately set out to disprove it. Indeed, the claim turns out to be false.

However, this suggestion coming from somebody as mathematically gifted as Sasha gave me the impression that perhaps there was a 'twisted version' of his proposed variant that really would be invariant. We then considered the quantity defined by $N_{i,j}(\underline{\alpha}) = \sum_{\underline{\epsilon} \in S_{i,j}(\underline{\alpha})} \prod_{k=1}^{m-2} \epsilon_k$. We showed that in case m is odd, this quantity is indeed independent of i and j. Moreover, $(-1)^{\#S(i,j)}$ is independent of i and j (this also holds for even m). In [1] Amnon and I gave three proofs for this, all in principle accessible to talented high school students. We were proud of the fact that our submitted version of the paper did not contain any reference whatsoever. Unfortunately the referee insisted that a reference to Gradshtyn and Ryzhik be included for the easy integral

$$\frac{1}{2\pi} \int_0^{2\pi} \cot(\frac{x}{2}) \sin(\beta x) = \mathrm{sgn}(\beta),$$

with β an integer. So this is how the paper ended up having one reference...

In Gijswijt and Moree [2] (a paper dedicated to the memory of Sasha) we prove a simple combinatorial principle which has the main result of [1] has a corollary. Several other examples of the combinatorial principle are given.

I am fully aware that the mathematical importance of the two papers described in this section is far less than that of the first section, but I had comparable fun in working on them and in both cases the initial discussion over tea with Sasha is still engraved in my memory.

3. Outside tea time

A few times I had contact with Sasha outside the institute. Also in these situations his originality shone through. Once the discussion, over dinner, was about the importance of making the right choice for a PhD subject. According to Sasha this was highly comparable with marrying the right wife. After all, one has to live at least for four years with a PhD topic, so it better be suitable. Moreover, just as one should avoid the temptation to marry a woman because she is rich and drives around in a flashy yellow Lamborghini, one should avoid the temptation to go for a PhD subject just because it is a popular/fashionable/hot topic at the time. The basic point is that, in the first place, one should love his PhD topic.

It was always my impression (but based on a modest amount of time spent together) that Sasha's ideal was to be a mathematical butterfly who with ease went from mathematician to mathematician to exchange ideas and carry out cross fertilisation and who did not want be (mathematically) confined in any sense and did his best to a 'one-of-a-kind' mathematician. On some moments I observed him when he was working alone he made the impression that behind this lightness projected to the outside world there was a much deeper, serious and very hard-working side to him.

In any case, to me, being much more of 'flower type of mathematician' than 'butterfly type', Sasha was and in my memory will always be the butterfly that

during two brief summers came along and made my 'flowery' existence more rosy and fruitful.

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