



Progress in Mathematics

Volume 260

Series Editors

H. Bass

J. Oesterlé

A. Weinstein

Arithmetic and Geometry Around Hypergeometric Functions

Lecture Notes of a CIMPA Summer School held at
Galatasaray University, Istanbul, 2005

Rolf-Peter Holzapfel
A. Muhammed Uludağ
Masaaki Yoshida
Editors

Birkhäuser
Basel • Boston • Berlin

Editors:

Rolf-Peter Holzapfel
Institut für Mathematik
Humboldt-Universität zu Berlin
Unter den Linden 6
D-10099 Berlin
e-mail: holzapfl@mathematik.hu-berlin.de

Masaaki Yoshida
Department of Mathematics
Kyushu University
Fukuoka 810-8560
Japan
e-mail: myoshida@math.kyushu-u.ac.jp

A. Muhammed Uludağ
Department of Mathematics
Galatasaray University
34357 Besiktas, Istanbul
Turkey
e-mail: muhammed.uludag@gmail.com

2000 Mathematics Subject Classification 14J10, 14J28, 14J15, 11F06, 11S80, 33C65, 22E40, 11F55, 11G15, 11K22, 33C70, 32Q30, 33C05

Library of Congress Control Number : 2006939568

A CIP catalogue record for this book is available from the Library of Congress, Washington D.C., USA

Bibliographic information published by Die Deutsche Bibliothek
Die Deutsche Bibliothek lists this publication in the Deutsche Nationalbibliografie;
detailed bibliographic data is available in the Internet at <<http://dnb.ddb.de>>.

ISBN 978-3-7643-8283-4 Birkhäuser Verlag AG, Basel – Boston – Berlin

This work is subject to copyright. All rights are reserved, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, re-use of illustrations, broadcasting, reproduction on microfilms or in other ways, and storage in data banks. For any kind of use whatsoever, permission from the copyright owner must be obtained.

© 2007 Birkhäuser Verlag AG, P.O. Box 133, CH-4010 Basel, Switzerland
Part of Springer Science+Business Media
Printed on acid-free paper produced of chlorine-free pulp. TCF ∞
Printed in Germany

ISBN-10: 3-7643-8283-X
ISBN-13: 978-3-7643-8283-4

e-ISBN-10: 3-7643-7449-7
e-ISBN-13: 978-3-7643-8284-1

9 8 7 6 5 4 3 2 1

www.birkhauser.ch

Preface

This volume comprises the Lecture Notes of the CIMPA Summer School *Arithmetic and Geometry around Hypergeometric Functions* held at Galatasaray University, Istanbul during June 13-25, 2005. In the Summer School there were fifteen lectures forming an impressive group of mathematicians covering a wide range of topics related to hypergeometric functions. The full schedule of talks from the workshop appears on the next page. In addition to the lecture notes submitted by its lecturers, this volume contains several research articles.

A group of forty graduate students and young researchers attended the school. Among the participants there were 2 Algerian, 3 American, 1 Armenian, 1 Bulgarian, 1 Canadian, 3 Dutch, 2 Georgian, 7 German, 1 Indian, 2 Iraqi, 1 Iranian, 1 Italian, 1 Russian, 5 Japanese, 23 Turkish and 1 Ukrainian mathematicians, including the lecturers.

We would like to thank the *Centre des Mathématiques Pures et Appliquées*, for their financial support and Professor Michel Jambu for organizational help. We could support participants from across the region thanks to the generous financial help provided by the *International Center for Theoretical Physics* (ICTP) and the *International Mathematical Union* (IMU). The local participants has been supported by the *Scientific and Technological Research Council of Turkey* (TÜBİTAK).

This summer school has been realized not only by financial support from its sponsors but also thanks to the generosity of its lecturers, who all agreed to finance their travel from their own personal grants. Some of them did so also for the accomodation.

The proposal for the AGAHF Summer School was submitted to CIMPA in February 2004. During the long preparatory process and during the summer school, Aysegül Ulus, Özgür Ceyhan, and Özgür Kişisel contributed at various levels to the organization. We are grateful to them.

Sabine Buchmann is a French artist living in Istanbul, who likes to draw Ottoman-style miniatures of the boats serving across the bosphorus; these boats are an inseparable part of the city panorama. When asked, she liked the idea of a boat full of mathematicians and drew it for the conference poster — with the names of all the lecturers hidden inside, written in minute letters. Her miniature helped us much in attracting the audience of the summer school.

We are thankful to the student team hired by the university comprising Anet İzmitli, Egemen Kirant, Günce Orman, Haris Saybaşı and Eylem Şentürk for turning this summer school into a pleasant experience.

Finally we would like to thank warmly Prof. Dr. Duygun Yarsuvat, the rector of the Galatasaray University for offering us the great location and financial support of the university.

The second named editor was supported by TÜBİTAK grant Kariyer 103T136 during the summer school and during the preparation of this volume.

Rolf-Peter Holzapfel, A. Muhammed Uludağ and Masaaki Yoshida, Editors

PROGRAM

Daniel Allcock: Real hyperbolic geometry in moduli problems

Igor Dolgachev: Moduli spaces as ball quotients (followed by Kondo's lectures)

Rolf Peter Holzapfel: Orbital Varieties and Invariants

Michel Jambu: Arrangements of Hyperplanes

A. Kochubei: Hypergeometric functions and Carlitz differential equations over function fields

Shigeyuki Kondo: Complex ball uniformizations of the moduli spaces of del Pezzo surfaces

Edward Looijenga: (first week) Introduction to Deligne-Mostow theory

Edward Looijenga: (second week) Hypergeometric functions associated to arrangements

Keiji Matsumoto: Invariant functions with respect to the Whitehead link

Hironori Shiga: Hypergeometric functions and arithmetic geometric means (followed by Wolfart's lectures)

Jan Stienstra: Gel'fand-Kapranov-Zelevinsky hypergeometric systems and their role in mirror symmetry and in string theory

Toshiaki Terada: Hypergeometric representation of the group of pure braids.

A. Muhammed Uludağ: Geometry of Complex Orbifolds

Alexander Varchenko: Special functions, KZ type equations, and representation theory

Jürgen Wolfart: Arithmetic of Schwarz maps (preceded by Shiga's lectures)

Masaaki Yoshida: Schwarz maps (general introduction)

Contents

Preface	v
<i>Daniel Allcock, James A. Carlson and Domingo Toledo</i> Hyperbolic Geometry and the Moduli Space of Real Binary Sextics ..	1
<i>Frits Beukers</i> Gauss' Hypergeometric Function	23
<i>Igor V. Dolgachev and Shigeyuki Kondō</i> Moduli of K3 Surfaces and Complex Ball Quotients	43
<i>Amir Džambić</i> Macbeaths Infinite Series of Hurwitz Groups	101
<i>Rolf-Peter Holzapfel</i> Relative Proportionality on Picard and Hilbert Modular Surfaces	109
<i>Anatoly N. Kochubei</i> Hypergeometric Functions and Carlitz Differential Equations over Function Fields	163
<i>Shigeyuki Kondō</i> The Moduli Space of 5 Points on \mathbb{P}^1 and K3 Surfaces	189
<i>Eduard Looijenga</i> Uniformization by Lauricella Functions — An Overview of the Theory of Deligne–Mostow	207
<i>Keiji Matsumoto</i> Invariant Functions with Respect to the Whitehead-Link	245
<i>Thorsten Riedel</i> On the Construction of Class Fields by Picard Modular Forms	273
<i>Hironori Shiga and Jürgen Wolfart</i> Algebraic Values of Schwarz Triangle Functions	287
<i>Jan Stienstra</i> GKZ Hypergeometric Structures	313
<i>A. Muhammed Uludağ</i> Orbifolds and Their Uniformization	373

<i>Masaaki Yoshida</i>	
From the Power Function to the Hypergeometric Function	407
<i>Celal Cem Sarıoğlu (ed.)</i>	
Problem Session	431