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Differential Equations with Symbolic Computation

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Preface

This book provides a picture of what can be done in differential equations with advanced methods and software tools of symbolic computation. It focuses on the symbolic-computational aspect of three kinds of fundamental problems in differential equations: transforming the equations, solving the equations, and studying the structure and properties of their solutions. Modern research on these problems using symbolic computation, or more restrictively using computer algebra, has become increasingly active since the early 1980s when effective algorithms for symbolic solution of differential equations were proposed, and so were computer algebra systems successfully applied to perturbation, bifurcation, and other problems. Historically, symbolic integration, the simplest case of solving ordinary differential equations, was already the target of the first computer algebra package SAINT in the early 1960s.

With 20 chapters, the book is structured into three parts with both tutorial surveys and original research contributions: the first part is devoted to the qualitative study of differential systems with symbolic computation, including stability analysis, establishment of center conditions, and bifurcation of limit cycles, which are closely related to Hilbert's sixteenth problem. The second part is concerned with symbolic solutions of ordinary and partial differential equations, for which normal form methods, reduction and factorization techniques, and the computation of conservation laws are introduced and used to aid the search. The last part is concentrated on the transformation of differential equations into such forms that are better suited for further study and application. It includes symbolic elimination and triangular decomposition for systems of ordinary and partial differential polynomials. A 1991 paper by Wen-tsün Wu on the construction of Gröbner bases based on Riquier–Janet's theory, published in China and not widely available to the western readers, is reprinted as the last chapter. This book should reflect the current state of the art of research and development in differential equations with symbolic computation and is worth reading for researchers and students working on this interdisciplinary subject of mathematics and computational science. It may also serve as a reference for everyone interested in differential equations, symbolic computation, and their interaction.

The idea of compiling this volume grew out of the Seminar on Differential Equations with Symbolic Computation (DESC 2004), which was held in Beijing, China in April 2004 (see <http://www-calfor.lip6.fr/~wang/DESC2004>) to facilitate the interaction between the two disciplines. The seminar brought together active researchers and graduate students from both disciplines to present their work and to report on their new results and findings. It also provided a forum for over 50 participants to exchange ideas and views and to discuss future development and cooperation. Four invited talks were given by Michael Singer, Lan Wen, Wen-tsün Wu, and Zhifen Zhang. The enthusiastic support of the seminar speakers and the

high quality of their presentations are some of the primary motivations for our endeavor to prepare a coherent and comprehensive volume with most recent advances on the subject for publication. In addition to the seminar speakers, several distinguished researchers who were invited to attend the seminar but could not make their trip have also contributed to the present book. Their contributions have helped enrich the contents of the book and make the book beyond a proceedings volume. All the papers accepted for publication in the book underwent a formal review-revision process.

DESC 2004 is the second in a series of seminars, organized in China, on various subjects interacted with symbolic computation. The first seminar, held in Hefei from April 24–26, 2002, was focused on geometric computation and a book on the same subject has been published by World Scientific. The third seminar planned for April 2006 will be on symbolic computation in education.

The editors gratefully acknowledge the support provided by the Schools of Science and Advanced Engineering at Beihang University and the Key Laboratory of Mathematics, Informatics and Behavioral Semantics of the Chinese Ministry of Education for DESC 2004 and the preparation of this book. Our sincere thanks go to the authors for their contributions and cooperation, to the referees for their expertise and timely help, and to all colleagues and students who helped for the organization of DESC 2004.

Beijing
May 2005

Dongming Wang
Zhiming Zheng

Contents

Symbolic Computation of Lyapunov Quantities and the Second Part of Hilbert's Sixteenth Problem	1
<i>Stephen Lynch</i>	
Estimating Limit Cycle Bifurcations from Centers	23
<i>Colin Christopher</i>	
Conditions of Infinity to be an Isochronous Center for a Class of Differential Systems	37
<i>Wentao Huang and Yirong Liu</i>	
Darboux Integrability and Limit Cycles for a Class of Polynomial Differential Systems	55
<i>Jaume Giné and Jaume Llibre</i>	
Time-Reversibility in Two-Dimensional Polynomial Systems	67
<i>Valery G. Romanovski and Douglas S. Shafer</i>	
On Symbolic Computation of the LCE of N -Dimensional Dynamical Systems	85
<i>Shucheng Ning and Zhiming Zheng</i>	
Symbolic Computation for Equilibria of Two Dynamic Models	109
<i>Weinian Zhang and Rui Yan</i>	
Attractive Regions in Power Systems by Singular Perturbation Analysis	121
<i>Zhujun Jing, Ruiqi Wang, Luonan Chen and Jin Deng</i>	
Algebraic Multiplicity and the Poincaré Problem	143
<i>Jinzhi Lei and Lijun Yang</i>	
Formalizing a Reasoning Strategy in Symbolic Approach to Differential Equations	159
<i>Shilong Ma</i>	
Looking for Periodic Solutions of ODE Systems by the Normal Form Method	173
<i>Victor F. Edneral</i>	

Algorithmic Reduction and Rational General Solutions of First Order Algebraic Differential Equations	201
<i>Guoting Chen and Yujie Ma</i>	
Factoring Partial Differential Systems in Positive Characteristic	213
<i>Moulay A. Barkatou, Thomas Cluzeau and Jacques-Arthur Weil</i>	
On the Factorization of Differential Modules	239
<i>Min Wu</i>	
Continuous and Discrete Homotopy Operators and the Computation of Conservation Laws	255
<i>Willy Hereman, Michael Colagrosso, Ryan Sayers, Adam Ringler, Bernard Deconinck, Michael Nivala and Mark Hickman</i>	
Partial and Complete Linearization of PDEs Based on Conservation Laws	291
<i>Thomas Wolf</i>	
CONSLAW: A Maple Package to Construct the Conservation Laws for Nonlinear Evolution Equations	307
<i>Ruo-Xia Yao and Zhi-Bin Li</i>	
Generalized Differential Resultant Systems of Algebraic ODEs and Differential Elimination Theory	327
<i>Giuseppa Carrá Ferro</i>	
On “Good” Bases of Algebraico-Differential Ideals	343
<i>Wen-tsün Wu</i>	
On the Construction of Groebner Basis of a Polynomial Ideal Based on Riquier–Janet Theory	351
<i>Wen-tsün Wu</i>	
Index	369