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New Trends in the Theory of Hyperbolic Equations

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Editors

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Editorial Preface

Hyperbolic partial differential equations describe phenomena of material or wave transport in the applied sciences. Despite of considerable progress in the past decades, the mathematical theory still faces fundamental questions concerning the influence of nonlinearities or multiple characteristics of the hyperbolic operators or geometric properties of the domain in which the evolution process is considered. The current volume is dedicated to modern topics of the theory of hyperbolic equations such as

evolution equations – multiple characteristics – propagation phenomena – global existence – influence of nonlinearities.

It is addressed both to specialists and to beginners in these fields. The contributions are to a large extent self-contained.

The first contribution is written by Piero D'Ancona and Vladimir Georgiev. Piero D'Ancona graduated in 1982 from Scuola Normale Superiore of Pisa. Since 1997 he is full professor at the University of Rome 1. Vladimir Georgiev graduated in 1981 from the University of Sofia. Since 2000 he is full professor at the University of Pisa.

The first part of the paper treats the existence of low regularity solutions to the local Cauchy problem associated with wave maps. This introductory part follows the classical approach developed by Bourgain, Klainerman, Machedon which yields local well-posedness results for supercritical regularity of the initial data. The nonuniqueness results are established by the authors under the assumption that the regularity of the initial data is subcritical. The approach is based on the use of self-similar solutions. The third part treats the ill-posedness results of the Cauchy problem for the critical Sobolev regularity. The approach is based on the effective application of the properties of a special family of solutions associated with geodesics on the target manifold.

The second contribution is written by Hideo Kubo and Masahito Ohta. Hideo Kubo graduated from Hokkaido University in 1996. Since 2003 he is associate professor at Osaka University. Masahito Ohta graduated from the University of Tokyo in 1996. Since 2003 he is associate professor at Saitama University.

Initially they consider in their contribution wave equations with small nonlinear perturbation. The problems of interest are local well-posedness in time, blow-up, asymptotic behaviour and existence of self-similar solutions. The main topic of their contribution is how the theory of wave equations is transferred to systems of nonlinear wave equations with different propagation speeds. They explain, in particular, how the fact of different propagation speeds can be utilized to the advantage of a detailed analysis of initial value problems. The systematic study of such systems is based on pointwise decay estimates for solutions of the Cauchy problem for inhomogeneous wave equations in L_∞ spaces with hyperbolic weights. The blow-up results are established by deriving lower estimates of this type.

Further fields of interest of H. Kubo are associated with the study of the system of elastic equations and the Maxwell system. Among other things, M. Ohta is also interested in issues of stability and instability of standing wave solutions for nonlinear Klein-Gordon equations.

The third contribution is written by Mitsuhiro Nakao.

He received his doctoral degree (Doctor of Science) in 1977 from Kyushu University and is full professor at the same university since 1976. He has been strongly interested in decay problems for the wave equation with various types of dissipations in bounded domains. He developed his own techniques, which many authors use today ("Nakao's inequality"). Since 1998 he focused his interest on the Cauchy problem and on initial-boundary value problems in exterior domains. His strategy is to combine the energy method with the geometry of the exterior domain. For non-trapping domains a restricted localized effective dissipation is employed. He has derived decay results with algebraic rates for local and total energies. Moreover, he found the critical order of nonlinearities in his models. Here the property of stability comes in. In the future M. Nakao wants to apply his knowledge of decay properties for nonlinear damped wave equations to the problems concerning global attractors. He also plans to consider related problems for nonlinear degenerate parabolic equations which is another field of his research interest.

The fourth contribution is written by Karen Yagdjian. Karen Yagdjian has received his Doctor of Physical and Mathematical Sciences degree from Moscow State University in 1990. Since 2004 he is assistant professor at the University of Texas-Pan American. His main interests are microlocal analysis and its application to partial differential equations.

The main goal of his contribution is to study the phenomenon of parametric resonance for wave map type equations. In particular, he is interested in the study of the influence of the oscillating behaviour of coefficients in t on the global existence of small data solutions. A special transformation reduces the Cauchy problem for the wave map type equations to linear Cauchy problems for the wave equation with a special constraint. To attack these Cauchy problems Floquet' theory, especially Borg's theorem for Hill's equation is used. The question for stability and instability is discussed in a systematic way. Other models with growing coefficients or stabilizing coefficients are treated in a similar fashion.

The fifth contribution is written by Massimo Cicognani and Luisa Zanghirati. Luisa Zanghirati graduated from Ferrara University in 1965 and is full professor there since 1985. Massiomo Cicognani graduated from Bologna University in 1983 and is full professor there since 1998.

Their contribution is devoted to local in time existence of smooth solutions for nonlinear degenerate hyperbolic problems. Different kinds of degeneracies are explored. On the one hand they consider weakly hyperbolic Cauchy problems (characteristics of constant multiplicity), thus Levi conditions of nonlinear type come into play. On the other hand degeneracies are produced by low regularity in time of the coefficients.

The goal of the authors is to present a unified approach for these problems. The main tools of the approach are an effective diagonalization procedure with regularized characteristic roots, an effective representation of commutators, a sharp Gårding's inequality for systems and a suitable transformation containing the loss of derivatives. The loss of derivatives is characteristic for degenerate hyperbolic problems. Moreover, they describe the propagation of analytic regularity of solutions.

Further interests of L. Zanghirati are questions concerning hypoellipticity, and M. Cicognani is interested in microlocal methods to describe the propagation of singularities for pseudodifferential operators.

The sixth contribution is written by Michael Dreher and Ingo Witt.

Michael Dreher graduated from the University of Freiberg in 1999. Since 2004 he is assistant professor at the University of Konstanz. Ingo Witt graduated from the University of Bonn in 1995. Since 2004 he has a DFG fellowship at the Imperial College London.

The goal of the authors is to derive sharp energy estimates for weakly hyperbolic Cauchy problems with finite time degeneracy of the coefficients at $t = 0$. From such estimates they obtain the precise loss of regularity that depends on the spatial variables. In the case of time-dependent coefficients they show an interesting relation to the theory of edge Sobolev spaces, a tool which is used for the study of differential operators on manifolds with singularities. In the general case (where the coefficients' depend on time–and spatial variables) the authors introduce Sobolev spaces of variable order.

The main step is to find the correct class of pseudodifferential symbols and to establish a pseudodifferential calculus which contains a symmetrizer. An application of a sharp Gårding's inequality gives rise to a sharp energy estimate. Sharpness is proved by using the method of Lyapunov functionals, where suitable estimates lead to an instability result.

We would like to thank all the referees for their valuable contribution in the evaluation process. We also wish to thank Dr. Jens Wirth (TU Bergakademie Freiberg) for the considerable effort he put into producing the final layout of this volume. Last not least, the editors would like to thank all the staff of Birkhäuser Publishing Company, in particular, Dr. Thomas Hempfling, for the pleasant cooperation.

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M. Reissig B.-W. Schulze