Walter Thirring

A Course in Mathematical Physics

3

Quantum Mechanics of Atoms and Molecules

Translated by Evans M. Harrell

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Preface

In this third volume of *A Course in Mathematical Physics* I have attempted not simply to introduce axioms and derive quantum mechanics from them, but also to progress to relevant applications. Reading the axiomatic literature often gives one the impression that it largely consists of making refined axioms, thereby freeing physics from any trace of down-to-earth residue and cutting it off from simpler ways of thinking. The goal pursued here, however, is to come up with concrete results that can be compared with experimental facts. Everything else should be regarded only as a side issue, and has been chosen for pragmatic reasons. It is precisely with this in mind that I feel it appropriate to draw upon the most modern mathematical methods. Only by their means can the logical fabric of quantum theory be woven with a smooth structure; in their absence, rough spots would inevitably appear, especially in the theory of unbounded operators, where the details are too intricate to be comprehended easily. Great care has been taken to build up this mathematical weaponry as completely as possible, as it is also the basic arsenal of the next volume. This means that many proofs have been tucked away in the exercises. My greatest concern was to replace the ordinary calculations of uncertain accuracy with better ones having error bounds, in order to raise the crude manners of theoretical physics to the more cultivated level of experimental physics.

The previous volumes are cited in the text as I and II; most of the mathematical terminology was introduced in volume I. It has been possible to make only sporadic reference to the huge literature on the subject of this volume—the reader with more interest in its history is advised to consult the compendious work of Reed and Simon [3].

Of the many colleagues to whom I owe thanks for their help with the German edition, let me mention F. Gesztesy, H. Grosse, P. Hertel, M. and T.

Hoffmann-Ostenhof, H. Narnhofer, L. Pittner, A. Wehrl, E. Weimar, and, last but not least, F. Wagner, who has transformed illegible scrawls into a calligraphic masterpiece. The English translation has greatly benefited from the careful reading and many suggestions of H. Grosse, H. Narnhofer, and particularly B. Simon.

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Contents

Symbols Defined in the Text

p, q	momentum and position coordinates	
ψ	Schrödinger wave function	
$\hbar$	Planck's constant	
$\mathbf{L}$	orbital angular momentum	
l	angular momentum quantum number	
Z	nuclear charge	
r_b	Bohr radius	(1.2.3)
Ry	Rydberg	(1.2.4)
$\mathbb{E}$	vector space	(2.1.1)
$\mathbb{C}$	set of complex numbers	(2.1.1)
$\Vert\ \Vert$	norm	(2.1.4)
$\Vert\ \Vert_p$	p-norm	(2.1.5; 6)
$L^p(K, \mu)$	space of p-integrable functions on K	(2.1.5; 6)
l^p	sequence space	(2.1.6; 2)
$\langle \vert \rangle$	scalar product	(2.1.7)
e_γ	basis vector	(2.1.12; 3)
$\mathbb{E}'$	dual space to $\mathbb{E}$	(2.1.16)
$\mathscr{L}(\mathbb{E}, \mathbb{F})$	space of continuous, linear mappings from $\mathbb{E}$ to $\mathbb{F}$	(2.1.24)
$\mathscr{B}(\mathbb{E})$	space of bounded operators on $\mathbb{E}$	(2.1.24)
a^*	adjoint operator for a	(2.1.26; 3)
w-lim, $\rightharpoonup$	weak limit	(2.1.27)
s-lim, $\to$	strong limit	(2.1.27)
lim, $\Rightarrow$	norm limit	(2.1.27)
l^0	sequence space	(2.2.2)
Sp(a)	spectrum of a	(2.2.13)
$a \geq b$	partial ordering of operators	(2.2.16)
$X(\mathscr{A})$	set of characters	(2.2.25)
$(\Delta_w(a))^2 = (\Delta a)^2 = \Delta a^2$	mean-square deviation	(2.2.33; 3)

$\mathscr{P}$	propositional calculus	(2.2.35)
$p_1 \wedge p_2$	intersection of propositions	(2.2.35(i))
$p_1 \vee p_2$	union of propositions	(2.2.35(ii))
$\sigma_x, \sigma_y, \sigma_z$	spin matrices	(2.2.37)
π	representation	(2.3.1)
$\mathscr{M}'$	commutant of $\mathscr{M}$	(2.3.4)
$\mathscr{Z}$	center	(2.3.4)
$\theta(x)$	step function	(2.3.14)
$\sigma_p(a)$	point spectrum	(2.3.16)
σ_{ac}	absolutely continuous spectrum	(2.3.16)
$\sigma_s(a)$	singular spectrum	(2.3.16)
σ_{ess}	essential spectrum	(2.3.18; 4)
$\mathrm{Tr}\, m$	trace of m	(2.3.19)
$\mathscr{C}_1$	trace-class operators	(2.3.21)
$\mathscr{C}_2$	Hilbert–Schmidt operators	(2.3.21)
$\mathscr{C}$	compact operators	(2.3.21)
$\mathbf{T}$	time-ordering	(2.4.10; 3)
$D(a)$	domain of definition of a	(2.4.12)
$\mathrm{Ran}(a)$	range of a	(2.4.12)
$\Gamma(a)$	graph of a	(2.4.15)
$a \supset b$	a extends b	(2.5.1)
$Q(q)$	quadratic-form domain	(2.5.17)
$\mathscr{W}$	Weyl algebra	(3.1.1)
$(z\|z')$	scalar product	(3.1.2; 1)
$\|l, m\rangle$	angular momentum eigenvectors	(3.2.13)
$L_\pm$	circular components of $\mathbf{L}$	(3.2.13)
$\mathrm{ad}_H^n(a)$	$(\text{derivation})^n$	(3.3.1)
P_{ac}	projection onto the absolutely continuous eigenspace	(3.4.4)
$\mathscr{A}$	algebra of asymptotic constants	(3.4.6)
$a_\pm$	limit of an asymptotic constant	(3.4.6)
$\tau_\pm$	homomorphism $\mathscr{A} \to \mathscr{A}_\pm$	(3.4.6)
$\Omega_\pm$	Møller operators	(3.4.7; 4)
P_α	projection for the channel with H_α	(3.4.17)
$Q_{\alpha\pm}$	channel decomposition of P_{ac}	(3.4.17)
$S_{\alpha\beta}$	S matrix in the interaction representation	(3.4.23)
$R(\alpha, z)$	resolvent	
$P_k(\alpha)$	projection operator for the perturbed Hamiltonian $H(\alpha)$	(3.5.1)
$t(k)$	t matrix	
$f(k; \mathbf{n}', \mathbf{n})$	angular dependence of the outgoing spherical wave	(3.6.10;3)
D	delay operator	(3.6.17)
$\sigma(\mathbf{k}, \mathbf{k}_0)$	differential scattering cross-section	(3.6.19)
σ_t	total scattering cross-section	(3.6.19)
a	scattering length	(3.6.23; 5)
$\mathbf{F}$	Runge–Lenz vector	(4.1.7)
A_k, B_k	generators of O(4)	(4.1.8)