Semiconductor Equations

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Preface

In recent years the mathematical modeling of charge transport in semiconductors has become a thriving area in applied mathematics. The drift diffusion equations, which constitute the most popular model for the simulation of the electrical behavior of semiconductor devices, are by now mathematically quite well understood. As a consequence numerical methods have been developed, which allow for reasonably efficient computer simulations in many cases of practical relevance. Nowadays, research on the drift diffusion model is of a highly specialized nature. It concentrates on the exploration of possibly more efficient discretization methods (e.g. mixed finite elements, streamline diffusion), on the improvement of the performance of nonlinear iteration and linear equation solvers, and on three dimensional applications.

The ongoing miniaturization of semiconductor devices has prompted a shift of the focus of the modeling research lately, since the drift diffusion model does not account well for charge transport in ultra integrated devices. Extensions of the drift diffusion model (so called hydrodynamic models) are under investigation for the modeling of hot electron effects in submicron MOS-transistors, and supercomputer technology has made it possible to employ kinetic models (semiclassical Boltzmann-Poisson and Wigner-Poisson equations) for the simulation of certain highly integrated devices.

The focus of this book is the presentation of the hierarchy of semiconductor models ranging from kinetic transport equations to the drift diffusion equations. Particular emphasis is given to the derivation of the models and the physical and mathematical assumptions used therefore. We do not go into the mathematical technicalities necessary for a detailed analysis of the models but rather sacrifice rigour for the sake of conveying the basic properties and features of the model equations. The mathematically interested reader is encouraged to consult the references for in-depth investigations of specific subjects.

We address applied mathematicians, electrical engineers and solid state physicists. The exposition is accessible to graduate students in each of the three fields. In particular, we hope that this book will be useful as a text for advanced graduate courses in this area and we urge students to work the

problems, which can be found at the end of each chapter, for a deeper penetration of the material.

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