INTERNATIONAL CENTRE FOR MECHANICAL SCIENCES

COURSES AND LECTURES - No. 92

EKKEHART KRÖNER
UNIVERSITY OF STUTTGART

STATISTICAL CONTINUUM MECHANICS

COURSE HELD AT THE DEPARTMENT
OF GENERAL MECHANICS
OCTOBER 1971

UDINE 1971

SPRINGER-VERLAG WIEN GMBH

Originally published by Springer-Verlag Wien-New York in 1972

ISBN 978-3-211-81129-0 ISBN 978-3-7091-2862-6 (eBook)

DOI 10.1007/978-3-7091-2862-6

PREFACE

The scientific branch of statistical continuum mechanics, as the name says, serves the purpose of treating statistical problems in continuum mechanics. The first important successes were achieved in the field of turbulence in the thierties of this century. In recent time the statistical continuum mechanics has experienced a large extension by including the complex field of materials of a heterogeneous constitution such as polycristalline aggregates, multiphase mixtures, composites etc.. It is the main aim of the statistical theory in this field to calculate the effective properties from the given properties of the constituents and from some information about their structural arrangement. Important progress has been made in the case of elastic materials. Here the names of Beran, Molyneux, Lomakin, Volkov, Klinskikh, McCoy, and Mazilu deserve particular mention.

The great fluctuations in stress and strain which accompany plastic deformation lead to the expectation that the final theory of plasticity, too, will contain statistical elements. Investigations in this field are still very preliminary, however. So it is not clear at all whether the statistical theory should be developed along the lines of approach to turbulence or whether it should be a statistics of dislocations which, as is well-known, are the fundamental sources of the stress and strain fluctuations.

In this course I give, after the introduction in chapter I, a somewhat restricted but, I hope, still sufficiently detailed outline of the mathematical theory of probability and statistics (chapter II). Today, all the concepts and notations presented here are standard, hence I have not tried to be particularly original in this part of the course. The systematics of chapter II and of chapter III in which the mathematical theory is combined with physical laws into the formulation of statistical continuum mechanics follow largely the excellent monograph of M. Beran "Statistical Continuum Theories", Interscience Publishers 1968. Throughout this course I have tried, as far as possible, to employ the notation of Beran.

This should facilitate for the reader of the present text the study of the more comprehensive presentation of Beran.

In chapter IV the field of statistical turbulence which may already be considered classical is outlined very briefly, just to accustom the reader to the statistical approach. It also provides provides an idea of the form a statistical physical theory may assume.

Chapter V is the central part of this text into which also some of my own original ideas have been embodied. In my opinion this chapter contains the most important results in the field. I mention the discovery of Beran and McCoy and of Mazilu, which shows that the macroscopic theory of linearly elastic heterogeneous materials is in some sense congruent to the non-local (linear) theory of elasticity. In this connection also the particular role played by the perfectly disordered material is pointed out. The theoretical concept of the perfectly disordered material was introduced by me when I was a research visitor at the Applied Science Division of Harvard University in 1967. I am very grateful to Professor B. Budiansky who at that time drew my attention to the just published work of Beran and Molyneux and who assisted me during the first difficult steps in this new area.

The dynamical theory of heterogeneous materials is much less developed than the statical theory. Hence I have omitted it completely and only pointed to certain additional difficulties which enter in the transition from statics to dynamics.

Chapter VI very briefly introduces a few remarks on stochastic elements which are inherent in the phenomenon of plasticity. These remarks are mainly meant to draw the attention of those interested in plasticity to a different kind of approach which could well develop into a new important branch of statistical continuum mechanics.

The field of turbulence, elasticity and plasticity are not the only fields within continuum mechanics where problems of a stochastic nature occur. I mention the area of flow through porous media which is treated by Beran. Stochastic processes certainly also take place in visco-elastic media. There does not seem to exist much work in this field. It is for the reason of compactness of this course that the present text has been confined to the theories of turbulence, and hetero-

geneous materials and a little bit of plasticity. I believe that inspite of this restriction the problems treated here form a rather representative cross section of the problems of continuum mechanics - or even of continuum physics in general - which require stat istical treatment.

It is a pleasure to thank Dr. B. K. D. Gairola for his thotough reading of the manuscript, Dr. J. Burbach, Dipl. Phys. J. Koch and Professor S. L. Koh for numerous discussions on the effective elastic moduli of heterogeneous matrials, Mrs. H. Wetzlberger for the careful and speedy typing of the manuscript and the Deutsche Forschungsgemeinschaft which has supported my own research in the field. Finally, I would like to thank all members of CISM for their great hospitality which made my stay at the Centre so pleasant.

Udine, October 1971

E. K.

Since this preface was written, a number of important new results in the field of interest have been obtained. It seems to me that the monograph will greatly gain in value if at least a brief report on such results were included. Beside a few corrections of the original text I, therefore, have inserted the section 5.20 and new references which lead the reader to the latest work in our area.

Stuttgart, January 1973

E. K.