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Daide Bigoni

*Editor*

# Extremely Deformable Structures



Springer

*Editor*

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## PREFACE

*Structures have traditionally been designed to work below their critical load, because any instability was normally identified as connected to failure or loss of functionality. Instability and bifurcation were viewed simply as potentially dangerous phenomena and hence structural deformations under load were required to be small. Recently, a variety of soft structures have been considered in mechanics. These are structures that work in a large deformation regime, where elastic elements are subject to extreme deformations and loads well beyond the critical values for buckling. Examples of structures that exhibit excellent mechanical performance even under severe deformation conditions are found in biological systems, deployable space structures, and a variety of devices in everyday use.*

*The lesson from nature is that the possibility of exploiting highly deformable structures, made for instance of rubber or gel, may open new and unexpected technological possibilities.*

*The exploration of these possibilities is the focus of this volume and of the so-called Extreme Mechanics, an emerging branch of the instability of solids and structures. This branch is aimed at the investigation of instabilities as related to pattern formation and the subsequent nonlinear behaviour of large deformations.*

*Here the challenge is the design of deformable and bi-stable mechanisms, which can give superior mechanical performance and which will have an impact on many high tech applications such as stretchable electronics, nanotube serpentines, deployable structures for aerospace engineering, cable deployment in the ocean, as well as on sensors and flexible actuators and vibration absorbers.*

*This monograph is the collection of the Lecture Notes for the Advanced School 'Extremely Deformable Structures' held at the International Centre for Mechanical Sciences (CISM) in Udine, Italy, June 2-6 2014. The course was given by six lecturers and attended by nearly fifty participants from eight European and four extra European countries. The chapters are devoted to an introduction to the methods used in the study of the stability of elastic structures in the finite dimensional case (A. Lazarus, C. Maurini and S. Neukirch), in the infinite dimensional case for the Euler elastica (D. Bigoni, F. Bosi, D. Misseroni, F. Dal Corso, and G. Noselli), and to the advanced problem*

*of the dynamics of a naturally curved elastica (B. Audoly, A. Callan-Jones, and P.-T. Brun). Two-dimensional problems are introduced with the purpose of analyzing the mechanics of two-dimensional periodic and highly deformable cellular structures (K. Bertoldi) and of the folding and deployment of thin-shells (S. Pellegrino). Structural concepts are linked to the modelling of growth processes occurring in biology (A. Erlich, Th. Lessinnes, D. E. Moulton, and A. Goriely). It is believed that the volume can represent a valid introduction to the field of extreme mechanics.*

*I wish to thank the Rectors of the CISM Professors E. Guazzelli, F. Pfeiffer, and F.G. Rammerstorfer, the Secretary General Professor B.A. Schrefler and all the staff for the warm hospitality and kind assistance during the course. Finally, I would like to gratefully acknowledge support from the FP7-PEOPLE-IDEAS-ERC-2013-ADG-340561-INSTABILITIES.*

*Davide Bigoni*

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