

Foundations of Engineering Mechanics

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Yury Vetyukov

Nonlinear Mechanics of Thin-Walled Structures

Asymptotics, Direct Approach and Numerical
Analysis

 Springer

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Preface

The recent progress in the theoretical mechanics of solids is often not regarded by the engineering community as a basis for the analysis of practical problems. Despite the high level of modern theories of thin-walled continua, the vast majority of numerical methods and solutions rest upon approximations of three-dimensional fields over the thickness. But “there is nothing more practical than a good theory” (attributed to L. Boltzmann), and the intent of this book is to bridge the gap between the theoreticians and the structural engineers. Modern methods of analysis contribute to the elegance and efficiency of the developed theoretical formulations, and finally, to the trustworthiness of numerical schemes. Their simplicity is demonstrated in the book by the source code for modeling complicated behavior of thin-walled structures, which is possible with modern high-level simulation environments such as Wolfram’s *Mathematica*.

The science of mechanics resides at the border between physics and mathematics. It has its own mentality and operates with its own criteria. The appropriate level of mathematical strictness is well demonstrated by the known joke:

A mathematician, a physicist, and an engineer were traveling through Scotland when they saw a black sheep through the window of the train. “Aha,” says the engineer, “I see that Scottish sheep are black.” “Hmm,” says the physicist, “You mean that some Scottish sheep are black.” “No,” says the mathematician, “All we know is that there is at least one sheep in Scotland, and that at least one side of that one sheep is black!”

Mathematics provides us with a handy toolbox for breaking new grounds, but experience shows that a physical way of thinking is often required for pioneering work in mechanics.

The scope of this book includes mechanical models of classical rods, plates, shells, and thin-walled rods of open profile, which are unified by the use of common methods of research. Classical theories of thin structures arise when the two ways of analysis meet and mutually complement each other. The procedure of asymptotic splitting in the three-dimensional model of the structure and the direct approach to an idealized dimensionally reduced continuum with the methods of Lagrangian mechanics constitute a very concise and formal method to developing geometrically

nonlinear theories with a high level of consistency. These analytical technologies play a central role in the theoretical parts of the book, which is counterbalanced by an extensive demonstration of possibilities of numerical analysis with the developed models. The presented material is self-sufficient, and the basic notions are discussed in the introductory part. Nevertheless, preliminary knowledge in the theory of elasticity, analytical mechanics and basic ideas of the method of finite elements should be recommended. Many theoretical and especially numerical aspects are illustrated by examples of mathematical modeling, performed with the *Mathematica* software. This modern language of science allows complicated simulations to be performed residing at the problem-oriented level without the need of programming sophisticated algorithms of numerical mathematics. A short reference for *Mathematica* is provided in Chap. 6. The source code of the simulations is an important constituent of the text of the book. It practically illustrates the proposed methods of modeling and provides the simulation results in their “naked” form, as nothing is hidden and everything can be reproduced. The files with these simulations are available for download at the SpringerLink online platform,¹ which grants the reader a possibility to experiment with the developed algorithms or to enhance them, avoiding the burden of retyping the necessary source code.

The author’s understanding and aesthetic feeling of mechanics were greatly influenced by the learning and work together with Prof. Vladimir Eliseev (Yeliseyev), who is still carrying the spirit of the school of mechanics, founded at the Polytechnic University of St. Petersburg (former Leningrad Polytechnic University) by Prof. Anatolii I. Lurie. To my father Prof. Mikhail Vetyukov I am obliged for the decision to choose mechanics for my studies and further work. Prof. Hans Irschik from the Johannes Kepler University Linz has greatly contributed to the present work with his vivid interest to the subject and many important comments, which helped improving the quality and readability of the text. I also express my gratitude to Prof. Alexander Belyaev from the Polytechnic University of St. Petersburg, as well as to Prof. Michael Krommer, Dr. Peter Gruber, Dr. Alexander Humer and other colleagues from the Johannes Kepler University Linz for important discussions and for their attention to the manuscript. This work has been supported by the Austrian COMET-K2 programme of the Linz Center of Mechatronics (LCM), and was funded by the Austrian federal government and the federal state of Upper Austria. I am very thankful to my mother Olga and my daughters Anastasia and Elena, who have been a source of inspiration and encouragement in my life and in writing this book.

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