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DAMAGE MECHANICS AND
MICROMECHANICS OF LOCALIZED
FRACTURE PHENOMENA IN
INELASTIC SOLIDS

EDITED BY

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PREFACE

This book resulted from a series of lecture notes presented in CISM, Udine. It has two major contributions: Damage Mechanics and Localization in Inelastic Deformation. The course is intended to provide researchers and graduate students with a clear and thorough presentation of the recent advances in continuum damage mechanics for both metals and metal matrix composites as well as the micromechanics of localization in inelastic solids.

Damage Mechanics and Applications: *The major goal of this part of the course is to present many of the different constitutive damage models that have recently appeared in the literature. Another goal is to clearly present the different approaches to this topic in a single complete course that is easily accessible to researchers and graduate students in civil engineering, mechanical engineering, engineering mechanics, aerospace engineering, and material science. The course material was delivered in well-organized lectures that started with the preliminaries and proceeded to advanced topics.*

Micromechanics of Localization in Inelastic Solids and Applications: *The main objective of this contribution is to discuss very efficient procedures of the numerical investigation of localized fracture in inelastic solids generated by impact-loaded adiabatic processes. Particular attention is focused on the proper description of a ductile mode of fracture propagating along the shear band for high impact velocities. This procedure of investigation is based on utilization the finite element or finite difference methods for regularized thermo-elasto-viscoplastic constitutive model of damage material using both rate dependency and non-local approaches.*

It is noteworthy to stress that all considered numerical examples are motivated by recent experimental observations. Qualitative comparison of numerical results with experimental observation data is presented. The numerical results obtained have proven the usefulness of the thermo-elasto-viscoplastic theory in the numerical investigation of dynamic shear band propagation and localized fracture using rate dependency and nonlocal formulations.

Invited Lecturers:

Tomasz Lodygowski (Poznan University of Technology, Poland), Piotr Perzyna (Institute of Fundamental Technological Research, Poland), Antonio Rinaldi (University of Rome – Tor Vergata – Italy), and George – Voyiadjis (Louisiana State University, USA).

Coordinator: *George – Voyiadjis*

The authors present in **Part I** the micro-mechanical damage model that accounts for the nonlocal microscopic interactions in the simulation of metal/composite impact and severe contact stress problems. This is achieved by introducing the contributions of damage and its corresponding gradients in the virtual power relations as measures of micro motion of damage within the bulk. By using these internal state variables together with displacement and temperature, the constitutive model is formulated with state laws based on the free energies and the complimentary laws based on the dissipation potentials.

In this work also the mechanics of small damage is presented using a consistent mathematical and mechanical framework based on the equations of damage mechanics. In this regard, the new scalar damage variable is investigated in detail. The investigation in this work has been carried out for seeking a physical basis is sought for the damage tensor \mathbf{M} that is used to link the damage state of the material with effective undamaged configuration. The approach presented here provides for a strong physical basis for this missing link. In particular, the authors have made an important link between the damage tensor and fabric tensors.

Computational aspects of the presented theory are also discussed. Numerical integration algorithms, verification and validation process of the theory are discussed. The finite element simulations are also performed by implementing the presented model in the commercial finite element code ABAQUS 6.8.3 as a user defined subroutine (VUMAT).

The outline of the material presented in **Part I** is as follows: in section 2, general mechanisms of the perforation and penetration mechanisms are discussed. A coupled rate-dependent (viscoplasticity) continuum damage theory is presented. In section 3, mechanics of small damage in fiber reinforced composite material is presented. In section 4, a comparative study has been made on the damage variables of the continuum. In section 5, computational aspects of the theory are discussed. The elastic predictor and coupled viscoplastic-viscodamage corrector algorithm that allows for total uncoupling of geometrical and material nonlinearities are presented. The nonlinear algebraic system of equations is solved by consistent linearization and the Newton-Raphson iteration. A derivation of a new definition for the consistent tangent stiffness matrix that is essential to maintaining the asymptotic quadratic rate of convergence is also presented in section 5. In section 6, numerical simulations of material instability are introduced in order to validate and test the proposed finite strain approach along with the proposed algorithm and its implementation in the ABAQUS finite element code. In section 7, various numerical examples are presented in order to validate the reliability and capability of the theory in simulating various impact and contact stress problems. Experimental comparisons of the adiabatic shear localizations between the proposed model simulations and other independent results are presented. Effect of initial temperatures and loading rates on the development of shear localizations is also

investigated in this section. Model capabilities are preliminarily illustrated for the dynamic localization of inelastic flow in adiabatic shear bands and compared with the experimental results of steel plates by deformable blunt projectiles at various impact speeds.

A handful of methodologies can be pursued in alternative to (and in combination with) continuum mechanics to develop advanced damage models that are intrinsically suited to address complex issues, such as strain localization phenomena and sample-size dependence of structural failure, which are intimately related to microscale phenomena. Statistical Damage Mechanics (SDM) is one of them.

*In **Part II** a brief discourse about advances in SDM introduces this methodology and shows its potential. In general, SDM is a multidisciplinary field that seeks to devise non-conventional approaches to fracture and damage by means of discrete damage models accounting for the essential microscale properties of a material or structural system. The ultimate goal is the development of multiscale methodologies that can reliably predict the materials macroscale response in consideration of the microstructural evolution caused by the damage process. Starting from the lower length scale characteristic of the microstructure, the SDM multiscale approach delivers a surprising amount of new insight and makes for a precious companion to the micromechanics methods discussed previously in the book. The bottom-up approach proves to be very effective in understanding the physics of the damage localization process. Although SDM is not yet a mature discipline, by now it has a clear footprint, after nearly two decades of research. This part of the book describes the state of the art and exposes recent trends in SDM for consideration and discussion by the larger solid mechanics community. Some of the concepts discussed herein □yet at their infancy □possess the potential to develop into full blown, innovative, and successful engineering tools. Designing the materials from the microstructure is already a current industry practice to achieve superior damage tolerance, partly driven by widespread usage of composites. SDM may help to do it even better. Another brand new application field for SDM is represented by nanotechnology, especially as far as the design of small (micro- and nano-sized) mechanical systems, such as MEMS and NEMS, where sample-size effects and discreteness are pervasive aspects of the design. After the introductory section 1, Part II unfolds a logical overview of concepts and solutions that begins with simpler one-dimensional models (i.e. the parallel bar system or PBS) in section 2 and ends with multidimensional systems (i.e. spring networks) in section 3. Great emphasis is placed on the derivation of physically-based definitions of the damage parameter for quasi-brittle systems that stem from a pervasive statistical rationale. In section 2, though, a coupled damage-plasticity model is also covered for ductile systems, limited to the one dimensional case. Finally, the last portion of section 3 focuses on sample-size effects in quasi-brittle materials and*

shows how to obtain and harness scaling laws into fractal-based constitutive relations for engineering applications.

The main objective of **Part III** is to show the broad application of the thermodynamic theory of elasto-viscoplasticity for the description of important problems in modern manufacturing processes, and particularly for meso-, micro-, and nano-mechanical issues. This description is particularly needed for the investigation by using the numerical methods how to avoid unexpected plastic strain localization and localized fracture phenomena in new manufacturing technology.

In the first part the development of thermo-elasto-viscoplastic constitutive model of a material which takes into consideration the induced anisotropy effects as well as observed contribution to strain rate effects generated by microshear banding is presented. Analysis of recent experimental observations concerning investigations of fracture phenomena under dynamic loading processes suggests that there are two kind of induced anisotropy: (i) the first caused by the residual type stresses produced by the heterogeneous nature of the finite plastic deformation in polycrystalline solids and (ii) the second called the fracture induced anisotropy generated by the evolution of the microdamage mechanisms. It is noteworthy to stress that both these induced anisotropy effects are coupled and this property has to be taken into account in the proposed constitutive description. On the other hand we very well know from recent experimental observations concerning investigation of dynamic loading processes that formation of microshear bands influences the evolution of microstructure of a material. The basic assumption is that microshear banding contributes to viscoplastic strain rate effects. The model is developed within the thermodynamic framework of a unique, covariance constitutive structure with a finite set of the internal state variables. A set of internal state variables consists of one scalar and two tensors, namely the equivalent inelastic deformation, the second order microdamage tensor with the physical interpretation that defines the volume fraction porosity and the residual stress tensor (the back stress). To describe suitably the influence of both induced anisotropy effects and the stress triaxiality observed experimentally the new kinetic equations for the microdamage tensor and for the back stress tensor are proposed. To describe the contribution to strain rate effects generated by microshear banding we propose to introduce certain scalar function which affects the relaxation time in the viscoplastic flow rule. The relaxation time is used as a regularization parameter. Fracture criterion based on the evolution of the anisotropic intrinsic microdamage is formulated. The fundamental features of the proposed constitutive theory have been carefully discussed.

The objective of the second part is to discuss very efficient procedure of the numerical investigation of localized fracture in inelastic solids generated by impact-loaded adiabatic processes. Particular attention is focused on the proper description of a ductile mode of fracture propagating along the shear band for

high impact velocities. This procedure of investigation is based on utilization the finite difference and finite element methods for regularized thermo-elasto-viscoplastic model of damaged material. The viscoplastic regularization procedure assures the stable integration algorithm by using the finite difference or finite element methods. Particular attention is focused on the well-posedness of the evolution problem (the initial-boundary value problem) as well as on its numerical solutions.

In **Part I** the behavior of selected brittle materials and structures (concrete and masonry) subjected to explosive loadings is discussed. For concrete the accepted Cumulative Fracture

Criterion (CFC) is exposed. It describes the degradation of the material under fast dynamic processes accompanied by the strong waves propagation phenomenon and large strain rates of deformation. To overcome the computational difficulty in the analyses of such complex problems, the sub-modeling technique as well as splitting of the calculations into two separate parts: analysis of acoustic wave in the air and the propagation of stresses in structures, are used. Some instructive numerical examples of concrete and masonry walls are in focus of the presentation. The numerical tools and computer simulations allow for proper estimation of the structures safety and for taking the design decisions on how to ensure their expected strength.

George V. Voyiadjis, Baton Rouge, 2010



*The authors dedicate this volume to the Memory of Dusan Krajcinovic
a Friend, Mentor and Inspirator*

CONTENTS

Part I: Consistent Non Local Coupled Damage Model and Its Application in Impact Response of Composite Materials	1
1 Prologue	3
2 Nonlocal formulation of the highvelocity impact induced damage	5
2.1 Physical foundation of penetration and perforation mechanisms	5
2.2 A coupled rate dependent (Viscoplastic) continuum damage theory	7
3 Mechanics of small damage in fiber reinforced composite materials	13
3.1 New scalar damage variable	15
3.2 Damage evolution	19
3.3 Mechanics of small damage	24
3.4 Overall and local approaches to damage in composite materials	27
3.4.1 Overall approach	28
3.4.2 Local approach	30
3.4.3 Equivalence of two approaches	33
4 A Comparative Study of Damage Variables in Continuum Damage Mechanics	34
4.1 Scalar damage variables	34
4.1.1 Hypothesis of elastic strain equivalence . .	36
4.1.2 Hypothesis of elastic strain energy equivalence	38
4.2 Other damage variable	40
4.3 Stress and strains	44
4.4 Tensorial damage variables	44
4.4.1 Hypothesis of elastic strain equivalence . .	46

4.4.2	Hypothesis of elastic strain energy equivalence	47
4.4.3	Isotropic elasticity	49
4.4.4	Plane stress	52
4.4.5	Plane strain	54
5	Computational aspects of the proposed theory	58
5.1	Numerical integration	58
5.2	Return mapping algorithm	59
5.3	Coupled viscoplastic damage corrector algorithms	61
6	Verification and validation of the theoretical formulation	65
6.1	Calibration of the metallic material constants and model parameters	66
6.2	Calibration of the composite material constants and model parameters	69
7	Application of the theory in studying high velocity contact problems	74
7.1	Simulation of experimental work on the study of friction between metallic surfaces	74
7.2	Shear localization in strip tension	81
7.3	Shear localizations in cylindrical hat-shaped samples and comparisons with experimental results	84
7.4	A blunt projectile impact in a target	86
7.5	A blunt projectile impact in a composite plate	90
8	Conclusions	91

Part II: Advances in Statistical Damage	
Mechanics (SDM): New Modeling Strategies	103
1 Introduction: from Continuum to Discrete Modeling	105
2 One-Dimensional Models: Parallel Bundles Systems (PBS)	112
2.1 Pure Damage: Quasi-Brittle PBS	113
2.2 Damage and Plasticity: Generalized PBS	118
2.2.1 Correlation between p_p and p_f	124
2.3 Final Thoughts about PBS Applications	125
3 Higher-Dimensional Models: Spring Networks	126
3.1 Lattice Representation and Model Definition . . .	128
3.2 Simulations Results from Tensile and Multiaxial Tests	132
3.3 Physical Foundations of \bar{D}	151
3.3.1 A Perspective from Thermodynamics . . .	151
3.3.2 Energetics of the Lattice in Uniaxial Tests	156
3.3.3 Rational Damage Model for Uniaxial Tests	166
3.3.4 Simplified Closed-Form Models of \bar{D} . . .	179
3.4 Sample-Size Effects	200
3.4.1 Hardening Phase	201
3.4.2 Softening Phase	204
3.4.3 Model Validation and Final Remarks . . .	209
4 More Related Work	212
5 Concluding Remarks	213
Appendix 1: Notes About Numerical Solution Schemes	214
Appendix 2: The One-way Correlation Hypothesis	218
References	221

Part III: Application of the Thermodynamical Theory of ElastoViscoplasticity in Modern Manufacturing Processes	225
1 Prologue	229
2 The Thermodynamical Theory of Elasto-Viscoplasticity	236
2.1 Physical Foundations	236
2.2 Experimental Motivation for Microshear Banding Effects	238
2.3 Experimental Motivation for Induced Anisotropy Effects	248
2.4 Kinematics of finite deformation and fundamental definitions	255
2.4.1 Fundamental measures of total deformation	255
2.5 Finite elasto-viscoplastic deformation	256
2.5.1 Rates of the deformation tensor	259
2.5.2 Rates of the stress tensors	260
2.5.3 Fundamental properties of the Lie derivatives	261
2.6 Thermo-Elasto-Viscoplasticity Constitutive Model	262
2.6.1 Constitutive postulate	262
2.6.2 Fundamental assumptions	263
2.6.3 Microshear bending effects	264
2.6.4 Constitutive assumption for the plastic spin	265
2.6.5 Anisotropic intrinsic microdamage mechanisms	265
2.6.6 Kinematic hardening	266
2.6.7 Thermodynamic restrictions and rate type constitutive relations	267
2.6.8 Fracture criterion based on the evolution of microdamage	268
2.7 Analysis of the Fundamental Features of the Model	268
2.7.1 Invariance with respect to diffeomorphism	268

2.7.2	Finite plastic deformation and plastic spin effects	268
2.7.3	Plastic non-normality	269
2.7.4	Softening effects generated by microdamage mechanisms	269
2.7.5	Anisotropic effects in microdamage mechanisms	269
2.7.6	Plastic deformation induced anisotropic effects	269
2.7.7	Thermomechanical couplings (thermal plastic softening and thermal expansion) . . .	270
2.7.8	Influence of stress triaxiality on the evolution of microdamage	270
2.7.9	Rate sensitivity	270
2.7.10	Length-scale sensitivity of the constitutive model	271
2.7.11	Dissipation effects	272
2.7.12	Dispersion effects	273
2.7.13	Regularization of the evolution problem . .	274
2.7.14	Synergetic effects generated by cooperative phenomena	274

3 Constitutive Modelling for Dynamic Cyclic

Loadings	275	
3.2	Experimental and physical motivations	275
3.2	Fundamental assumptions	280
3.3	Intrinsic micro-damage process	280
3.4	Kinematic hardening	282
3.5	Thermodynamic restrictions and rate type constitutive relations	284
3.6	Fracture criterion based on the evolution of porosity	285
3.7	Rate independent plastic response	285
3.7.1	Rate independent plastic response as a limit case	285

3.7.2	Rate independent intrinsic micro-damage process	286
3.7.3	Fundamental rate type constitutive equations	287
4	Localization and Localized Fracture Phenomena in Cyclic Dynamic Loading Processes	289
4.1	Formulation of the evolution problem	289
4.2	Application of finite difference method	290
4.3	Stability criterion and the Lax-Richtmyer equivalence theorem	293
4.4	Identification procedure	293
4.4.1	Assumptions of the material functions	293
4.4.2	Identification of the material constants	295
4.4.3	Investigation of stability and convergence	296
4.4.4	Investigation of localization and fracture phenomena	297
4.5	Particular example	297
4.5.1	Dynamic, adiabatic, cyclic loading processes for a thin plate with sharp notch	297
4.5.2	Discussion of localization and localized fracture phenomena	298
5	Numerical Analysis of Macrocrack Propagation along a Bimaterial Interface under Dynamic Loading Processes	310
5.1	Experimental motivation	310
5.2	Thermo-elasto-viscoplastic model of a material	312
5.3	Identification procedure	314
5.3.1	Assumption of the material functions for an adiabatic process	314
5.3.2	Assumption of the material functions for an isothermal process	315
5.3.3	Determination of the material constants	316

5.4	Numerical solution of the initial boundary-value problems	318
5.4.1	Formulation of the initial boundary-value problem	318
5.4.2	Investigation of the macro crack propagation along a bimaterial interface	320
5.4.3	Influence of temperature effects for an adiabatic process	329

6 Numerical Investigation of Dynamic Shear Bands in Inelastic Solids as a Problem of Mesomechanics 332

6.1	Physical and experimental motivation	332
6.1.1	Analysis of mesomechanical problems	332
6.1.2	Experimental investigation of the initiation and propagation of shear bands	333
6.1.3	Fracture phenomena along localized shear bands	335
6.1.4	Temperature measurement	337
6.1.5	Possible shear band branching	338
6.2	Formulation of the evolution problem	338
6.2.1	Thermodynamic theory of elasto-viscoplasticity	338
6.2.2	Initial-boundary value problem (evolution problem)	340
6.2.3	Numerical solution of the evolution problem	340
6.3	Identification procedure	341
6.3.1	Assumption of the material functions for an adiabatic process	341
6.3.2	Determination of the material constants	341
6.4	Numerical examples	347
6.4.1	Propagation of shear bands	347
6.4.2	Discussion of the numerical results	347

7	Numerical Investigation of Localized Fracture Phenomena in Inelastic Solids	352
7.1	Thermo-elasto-viscoplastic model of a material . . .	352
7.2	Formulation of the evolution problem	352
7.2.1	Initial-boundary value problem (evolution problem)	352
7.2.2	Numerical solution of the evolution problem	353
7.3	Identification procedure	353
7.3.1	Assumption of the material functions for an adiabatic process	353
7.3.2	Determination of the material constants .	355
7.4	Numerical example. Investigation of localized fracture phenomena	356
7.4.1	Application of a finite difference method .	356
7.4.2	Numerical solution of the initial boundary-value problem	357
7.4.3	Discussion of the numerical results	363
8	Epilogue	365
	References	368

Part IV: Safety of Concrete and Masonry Structures under Unusual Loadings	377
1 Some remarks on damage and localized fracture	379
2 Introduction	381
3 Cumulative Fracture Criterion for brittle materials	382
3.1 Dynamic concrete behavior in codes	382
3.2 Cumulative Fracture Criterion (CFC)	383
3.3 Identification and the influence of used parameters	385
4 Numerical simulation	388
4.1 One-dimensional verification	388
4.2 Explosive loading	393
4.3 Three-dimensional verification for circular concrete plate	398
5 Engineering structure subjected to explosion	399
5.1 Pressure distribution in the air	399
5.2 Pressure distribution on the wall front surface . .	401
6 Conclusions	405
Bibliography	410