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POLY-, QUASI- AND RANK-ONE
CONVEXITY IN APPLIED MECHANICS

EDITED BY

JÖRG SCHRÖDER

UNIVERSITY DUISBURG-ESSEN, ESSEN, GERMANY

PATRIZIO NEFF

UNIVERSITY DUISBURG-ESSEN, ESSEN, GERMANY

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PREFACE

Many mechanical applications are associated to “generalized” convexity conditions, e.g. the modeling of fracture and self contact, the status of elasticity with respect to atomistic models, the understanding of microstructure induced by phase transformations, the passage from three-dimensional elasticity to models of rods and shells, applications in the field of biomechanics, carbon nanotube modeling, and finite-element formulation of nematic liquid crystal elastomers.

Related to these problems are the conditions of polyconvexity (Ball 1977), quasiconvexity (Morrey 1952) and rank-one convexity (Legendre-Hadamard ellipticity). In contrast to isotropic models the construction of anisotropic polyconvex functions remains an open field of research and has been treated in the course. Some well-known material models do not fulfill the quasiconvexity inequality. In these cases the construction of quasiconvex hulls may be advisable. Applications have been discussed for the St. Venant-Kirchhoff model and for nematic liquid crystals. Furthermore, focussing on material models satisfying the Legendre-Hadamard condition, the construction of rank-one convex functions is another important strategy.

The CISM course on “Poly-, Quasi- and Rank-One Convexity in Applied Mechanics”, held in Udine from September 24 to September 28, 2007, was addressed to master students, doctoral students, post docs and experienced researchers in engineering, applied mathematics and science who wished to broaden their knowledge in generalized convexity conditions and their impact in applied mechanics, particularly with regard to the constitutive modeling of complex material behavior as well as on the consequences of “validity” (existence) of solutions obtained within direct variational methods.

It is our pleasure to thank the lectures of the CISM course and contributors to this CISM lecture notes Sir John Ball (Oxford), Antonio DeSimone (Trieste), Annie Raoult (Paris), Miroslav Šilhavý (Prague), David J. Steigmann (Berkeley), as well as the additional contributors Daniel Balzani (Essen) and Vera Ebbing (Essen). Finally, we thank the 59 participants from 13 countries who made the course a success. We extend our thanks to the Rectors, the Board, and the staff of CISM for the excellent support and kindful help.

Jörg Schröder and Patrizio Neff

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