

Volker Achim Weberruß

**Universality in Statistical  
Physics and Synergetics**

Volker Achim Weberruß

# **Universality in Statistical Physics and Synergetics**

**A Comprehensive Approach  
to Modern Theoretical Physics**

with 38 Figures



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## Preface

This book is based on my research work I did between 1986 and 1992 at the *Institut für Theoretische Physik und Synergetik, Universität Stuttgart*. It might be of interest to all students and scientists who are interested in modern mathematical physics. It deals with one of the essential topics of modern physical research, namely the problem of universal aspects in statistical physics, and in doing so multi-component systems, like thermodynamic systems, laser systems and high-dimensional quantum systems, are considered. Synergetic aspects of multi-component systems will be discussed. Remarks to biological systems will be made, in particular remarks to the problem of EEG analysis. Non-linear aspects will be discussed. The problem of a macroscopic access to multi-component systems was an essential part of my research work. In this context it has to be emphasized that in this book the concept of hyper-surface equations will be introduced, which is a totally analytical strategy to determine distribution function parameters in an exact way by using only given measurement quantities. This concept can be used to determine both distribution functions of macroscopic systems and path integrals of quantum systems. In this context the property *self-similarity* will play a crucial role. A special extreme principle, the maximum information entropy principle, can be taken as a basis to introduce this concept. This principle will be discussed. In order to determine the dynamic behavior of multi-component systems, special evolution equations are needed. Such evolution equations will be discussed. In particular, equations of the *Langevin* type, the *Fokker-Planck* type and the *Schrödinger* type will be considered, and it will be shown that an elementary access to both equations of the *Fokker-Planck* and the *Schrödinger* type is possible. A special chapter will consider quantum system theory, in particular the concept of *Feynman* path integrals. In this context a general concept to calculate *Feynman* kernels will be introduced. An additional possibility to describe the behavior of multi-component systems is given by information theory. The problem of description of such systems with the tools of information theory will also play a crucial role in this book. It will be discussed that the concept of information theory allows a universal access to arbitrary multi-component systems. Additionally, basics of a relativistic system theory will be considered. In this context a connection of quantum systems and cosmological systems will be worked out. Then it will be shown that a universal access to all physical systems discussed in this book is possible by introducing the notion *Riemann universe*.

Due to the fact that physical systems of the micro-world, of the direct observable surrounding and cosmological systems are considered, and due to the fact that universal properties will be discussed, the structure of this book can be called *comprehensive* or *holistic*.

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