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Shapes and Diffeomorphisms

Second Edition

Springer

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To Geneviève, Hannah, Salomé, Simon

Introduction to the Second Edition

Besides correcting some of the typos and mistakes from the first edition and implementing a few changes to the notation to make it uniform, several significant additions appear in the second edition. Most notable is the introduction of a discussion of optimal control theory in an infinite-dimensional framework (Appendix D), which is then used in multiple places to enrich the presentation of diffeomorphic matching, and a new chapter on shape datasets (Chap. 13 of the second edition). A few other changes have been made. These are listed below, where chapters are referred to as x/y, the first and second numbers indicating the first and second editions, respectively.

- Chapter 1/1: The presentation of closed curves and integrals along them has been revised. Some sections have been reordered to improve readability.
- Chapter 2/2: No significant changes.
- Chapter 3 of the first edition has been removed.
- Chapter 4/3: No major changes were made. The material on discrete surfaces was moved to Chap. 5/4, in which a new discussion is added on consistent discrete-to-continuous approximation.
- Chapters 6/5 and 7/6 only had minor modifications.
- Chapter 8/7: A few additional results on compositions of diffeomorphisms and their derivatives have been included. Small modifications were made to the rest of the chapter, which also relies more directly on results from the appendix.
- Chapter 9/8: The discussion on invariant operators and kernels has been updated and clarified.
- Chapter 10/9: Only minor changes were made in most sections. Exceptions are (1) curve and surface matching, in which a discussion of varifold distances was added, and (2) frame matching, which has been rewritten with a slightly different action, and a more rigorous handling of the supports of the frame fields.
- Chapter 11/10: Some of the discussion now directly uses results from the appendix on ODEs and optimal control. The proof of the existence and uniqueness of solutions of the EPDiff equation has also been rewritten.
- Chapter 12/11 had only minor changes.

- Chapter 13/12 on metamorphosis has been extended, with more examples, including in particular a detailed discussion of metamorphosis applied to the tangent representation of curves.
- Chapter 13 in the second edition is new.
- Only minor changes were made in Appendix A.
- Appendix B was extended with a discussion of differential forms and some consequences of Stokes's theorem.
- Appendix C has essentially the same content, even though part of the write-up was revised.
- Appendix D now includes an introduction to optimal control theory.
- Appendices E and F were only slightly modified.

Introduction to the First Edition

Shape is a fascinating object of study. Understanding how a single shape can incur a complex range of transformations, while defining the same perceptually obvious figure, entails a rich and enticing collection of problems at the interface between applied mathematics, statistics, and computer science. Various applications in computer vision, object recognition, and medical imaging bring additional motivation for researchers to develop adequate theoretical background and methodology for solving these problems.

This book is an attempt at providing a description of the large range of methods that have been invented to represent, detect, or compare shapes (or more generally, deformable objects), together with the necessary mathematical background that they require. While certainly being a book on applied mathematics, it is also written in a way that will be of interest to an engineering- or computer-science-oriented reader, including in several places concrete algorithms and applicable methods, including experimental illustrations.

This book starts with a discussion of shape representation methods (Chapters 1–4), including classical aspects of the differential geometry of curves and surfaces, but borrowing also from other fields that have positively impacted the analysis of shape in practical applications, such as medial axes and discrete differential geometry.

The second part (Chapters 5–7) studies curve and surface evolution algorithms and how they relate to segmentation methods that can be used to extract shapes from images, using active contours or deformable templates. A reader with enough background in differential geometry may start reading this book at Chapter 6 or at Chapter 7 if the main focus of interest is on diffeomorphic registration and comparison methods.

In Chapters 7 and 8, basic concepts related to diffeomorphisms are introduced, discussing in particular how using ordinary differential equations associated with vector fields belonging to reproducing kernel Hilbert space provides a computationally convenient framework to handle them. Chapters 9 and 10 then focus on the registration of deformable objects using diffeomorphisms; in Chapter 9, we catalog a large collection of deformable objects and discuss matching functionals that can be used to compare them. Chapter 10 addresses diffeomorphic matching and focuses in particular on methods that optimize a matching functional combined with a regularization term that penalizes the distance of a diffeomorphism to the identity within the group.

The next two chapters (11 and 12) discuss metric aspects of shape analysis, with a special focus on the relation between distances and group actions. Both the global and infinitesimal points of view are presented. The classical Kendall's metric over configurations of labeled points is included, as well as a short discussion of Riemannian metrics on plane curves. Chapter 12 provides a presentation of the theory of metamorphosis. Chapter 13 provides an introduction to the statistical analysis of shape data.

In the appendices are provided fundamental concepts that are needed in order to understand the rest of this book. The main items are some elements of functional analysis (Appendix A), of differential and Riemannian geometry (Appendix B), and of ordinary differential equations (Appendix C). Appendix D provides an introduction to optimization and optimal control. Appendix E focuses on principal component analysis and Appendix F on dynamic programming. In all cases, the appendices do not provide a comprehensive presentation of these theories, but simply what is needed in the particular context of this book.

Chapters 1 to 5, which are (with a few exceptions) rather elementary, provide an introduction to applied differential geometry that is suitable for an advanced undergraduate class. They can be combined with Chapter 6 to form a graduate-level class on the same subject. The first six chapters are written (with a few exceptions) in order to be accessible without using the more advanced features developed in the appendices. Chapters 8 to 13 represent specialized, advanced graduate topics.

I would like to thank my students and collaborators, who have helped to make the ideas that are developed in these notes reach their current state of maturation. I would like, in particular, to express my gratitude to Alain Trouvé and Michael Miller, whose collaboration over the last decade has been invaluable. Special thanks also to Darryl Holm, David Mumford, and Peter Michor. This book was written while the author was partially supported by the National Science Foundation, the National Institute of Health, and the Office for Naval Research.

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