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Yang Yu

Orbital Dynamics in the Gravitational Field of Small Bodies

Doctoral Thesis accepted by
Tsinghua University, Beijing, China

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*To my parents, who supported me at each
step of the way; And to my wife, who
accompanied me writing this thesis.*

Supervisor's Foreword

One of the core issues in modern celestial mechanics is the orbital dynamics in the near-regime gravitational fields of Solar System small bodies, which is related to the mathematical nature of a class of nonlinear systems, and plays a critical basis for in situ explorations of different scientific goals. Lots of efforts have been made to reveal the characteristics of orbital motion in the vicinities of asteroids, and to improve the skills of asteroid research in methodology. This impressive Ph.D. thesis focuses on the rich dynamics associated with a tracer particle orbiting around the arbitrarily shaped asteroid, which is modelled in great detail. As an exploration to this astrophysical frontier, the author, Dr. Yang Yu, mastered advanced topics in nonlinear dynamics, asteroid mission design and asteroid system formation. It is remarkable the author adopted both precise numerical models and theoretical qualitative analysis in this work, and this should be highlighted as a valuable exploration to the issues in the modern celestial mechanics, which has been in the face of increasing complexity and resolution today.

The main innovative work of this thesis: the author exposed the 3D structure of the zero-velocity surface of a specific asteroid, and explored the stabilities of its equilibrium points; an efficient numerical method was proposed for hierarchical search of large-scale periodic orbits, and by applying this methodology, 29 periodic families were found about the given asteroid, based on which the author further discussed the topological transitions and the general motion patterns around these periodic orbits; the study of resonant orbits near the equatorial plane presented the essence of 1:1 resonance and the distribution of the resonant orbits; and in the study of the free motion near the asteroid surface, the thesis demonstrated how a detailed mechanical model may provide a feedback to our understanding of the coupled motion of the granular over the surface of a real asteroid.

This thesis was awarded 2014 Excellent Doctoral Dissertation of Tsinghua University, and got highly praised by the four reviewers, who concluded that this is a high-quality Ph.D. thesis, showing original results of academic significance and application value. I have benefited greatly from Dr. Yu's insights during our collaboration, and I hope you will share my experience after reading this book.

Beijing, China
February 2016

Hexi Baoyin

Preface

The orbital dynamics in the near-regime gravitational fields of Solar System small bodies (SSSB) is an important aspect of modern celestial mechanics, which is of abundant physical phenomena and offers insights into the mathematical expressions of astronomical events. During last two decades, several deep space probes have been launched for in situ explorations to these small worlds, and the orbital dynamics around a small body comes as one of the biggest challenges in space engineering. As an application of basic research, the work advanced in this thesis is about the common issues in the orbital dynamics around SSSBs, using high-resolution models, which serves as a bridge to the understanding of the orbital motion in the vicinity of a real small body.

Four types of orbits are discussed in this thesis: equilibrium points, periodic orbits, resonant orbits near the equatorial plane, and natural motion close to the surface. Specific asteroids' models are employed in these studies, and new algorithms are developed based on the polyhedral models, i.e. the Hierarchical Grid Search Method (HGSM) designed for searching the large-scale periodic orbits around a small body, and the surface mitigation model in order to mimic the complicated motion of a particle close to the surface. FORTRAN packages are developed for numerical implementation of these algorithms.

In the studies of equilibrium points and periodic orbits, we focus on the qualitative properties of the system, especially for the general behaviours of vicinal orbits. Four equilibrium points of asteroid 216 Kleopatra are exposed by checking the 3D geometries of the zero-velocity surfaces, and then their stabilities and topologies are determined. The general motion around the equilibrium points are decomposed into three types of local invariant manifolds, sketching out the general behaviours of nearby orbits. Six continuous families of local periodic orbits are obtained in the centre manifolds. In the study of large-scale periodic orbits, 29 new families around Kleopatra are generated using HGSM. Poincaré mapping is introduced to investigate the stability of the 29 families, and these families are classified into different types based on their topologies. It is noticed that the transition within the same family follows specific strategies, which characterizes the

topological evolution of the periodic orbits. Motions around the orbits of the 29 families are attributed to five simple patterns, and the general motion near the periodic orbits is qualitatively determined.

In the studies of resonant orbits near the equatorial plane and free motion close to the surface, we highlight the role of numerical experiments. The variation of orbital energy is analyzed to understand the dynamical nature of the 1:1 resonance. Grid search on the parameter space reveals the condition of this type of resonance. Observing that 1:1 resonance is the major cause of ejecting motions, we present the distribution of ejecting orbits around Kleopatra on the parameter space and determine the critical conditions. A high-risk region for the probes is found near Kleopatra for the rich ejecting orbits in the equatorial plane. The surface mitigation method is applied to the study of surface motion on asteroid 1620 Geographos. The global surface environment is evaluated, showing the connections between the free motions and the surface local geometries. Monte Carlo simulations are performed to investigate the trajectories initialized close to the surface. The results show that the free motions close to the surface are highly influenced by the local terrain, and several mechanisms may govern the free motion of different processes.

Noticing that most of this work is based on a specific small body, we generalize the results as consultative for similar types of issues. Essentially, the topics investigated in this thesis are common and representative for a large group of small bodies, and the ideas and approaches proposed here are supposed to be generic and portable.

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I would like to express special appreciation to my advisor Prof. Hexi Baoyin, who has been a tremendous mentor, always encouraging my work and allowing me to grow as a research scientist. I would also like to thank Prof. Junfeng Li, Prof. Tianshu Wang, Dr. Shengping Gong and Dr. Fanghua Jiang for the suggestions and discussions on my Ph.D. research. And thanks for the support of the National Basic Research Program of China (973 Program, 2012CB720000) and the National Natural Science Foundation of China (No. 11372150).

Contents

1	Introduction	1
1.1	Background	1
1.2	Academic Merits and Significance	3
1.3	Research Status Survey	5
1.3.1	Progress in Astronautic Dynamics	5
1.3.2	Progress in Planetary Science	8
1.3.3	Progress in Nonlinear Dynamics	10
1.4	Content and Innovations	12
1.4.1	Content of the Thesis	12
1.4.2	Innovations of the Thesis	13
	References	14
2	Modelling Orbital Dynamics in the Potential of Small Bodies	19
2.1	Introduction	19
2.2	Mechanical Environment in the Vicinity of Small Bodies	20
2.3	Descriptions of the Gravitational Field	24
2.3.1	Mass Point Cluster	25
2.3.2	Spherical/Ellipsoidal Harmonics	26
2.3.3	Polyhedral Method	29
2.4	Motion Equations	31
2.5	Dynamical Factor κ	33
2.6	Summary	36
	References	37
3	Stability of Equilibrium Points and Behaviour of Nearby Trajectories	39
3.1	Introduction	39
3.2	Equilibrium Points of Asteroid 216 Kleopatra	40
3.2.1	Equilibrium Points	40
3.2.2	Zero-Velocity Surface	43

3.3	Stability of the Equilibrium Points	45
3.4	Structure of Local Manifolds.	47
3.4.1	Three Types of Invariant Manifolds at E_1 and E_2	48
3.4.2	Three Types of Invariant Manifolds at E_3 and E_4	50
3.4.3	Periodic Orbital Families Determined from Central Manifolds.	51
3.5	Orbital Behaviours in the Neighbourhood of Equilibrium Points.	54
3.5.1	Behaviours of Trajectories Near E_1 and E_2	56
3.5.2	Behaviours of Trajectories Near E_3 and E_4	57
3.6	Summary	59
	References	59
4	Topological Classification and Stability of Large-Scale Periodic Orbits	61
4.1	Introduction	61
4.2	Periodic Orbits Around 216 Kleopatra	62
4.2.1	Hierarchical Grid Search Method	63
4.2.2	Families Generated Around Kleopatra	66
4.3	Stability of Families 1–29.	71
4.4	Topological Classification of Periodic Orbits.	74
4.5	Behaviours of Trajectories Near Periodic Orbits of 216 Kleopatra.	76
4.5.1	Periodic Orbits of Complex Saddle Type	78
4.5.2	Periodic Orbits of Real Saddle Type	80
4.5.3	Periodic Orbits of Central Saddle Type	80
4.5.4	Periodic Orbits of General Centre Type	81
4.6	Summary	82
	References	82
5	Orbital Resonance Near the Equatorial Plane of Small Bodies.	85
5.1	Introduction	85
5.2	1:1 Resonant Orbits Near the Small Bodies	86
5.2.1	Energy Equations	87
5.2.2	A Mechanism of 1:1 Resonance	88
5.2.3	Typical Resonant Orbits.	90
5.3	Parameter Dependence of the Resonance	90
5.4	Resonant Orbits Near the Equatorial Plane of 216 Kleopatra.	93
5.4.1	Parameter Conditions for Orbital Resonance.	94
5.4.2	Distribution of the Ejecting Orbits.	94
5.5	Summary	97
	Reference.	98

- 6 Natural Motion Near the Surface of Small Bodies 99**
 - 6.1 Introduction 99
 - 6.2 Modelling Mechanics of the Surface 100
 - 6.2.1 Modified Polyhedral Method 100
 - 6.2.2 C^1 Surface Interpolation over the Polyhedron 102
 - 6.2.3 Equation of Motion on Surface 104
 - 6.3 Modelling the Natural Motion Near the Surface 106
 - 6.3.1 Triggering Events Within Orbital Motion Segments 106
 - 6.3.2 Triggering Events Within Surface Motion Segments 108
 - 6.3.3 Global Trajectory Patching Technique 109
 - 6.4 Verification and Assessment 111
 - 6.4.1 Example 1: Gravitational Field of a Solid Sphere 111
 - 6.4.2 Example 2: Demonstration Trajectories over Different Bodies 112
 - 6.5 Analysis of the Natural Motion Near Asteroid 1620 Geographos 114
 - 6.5.1 Surface Slope and Equilibrium Area 114
 - 6.5.2 Surface Curvature and Liftoff Speed 115
 - 6.5.3 Behaviours of Global Trajectories over 1620 Geographos 117
 - 6.6 Summary 118
 - References 119
- 7 Conclusions and Future Directions 121**
 - 7.1 Conclusions 121
 - 7.2 Future Directions 122

Acronyms and Symbols

Acronyms

CRTBP	Circular Restricted Three-Body Problem
SSSB	Solar System Small Body
NEA	Near-Earth Asteroid
NASA	National Aeronautics and Space Administration
ESA	European Space Agency
AU	Astronomical Unit

Main Symbols

\mathbf{r}, r	Position
\mathbf{a}, a	Acceleration
$\boldsymbol{\omega}, \omega$	Angular velocity
L	Orbital momentum of moment
x	State variable
M	Mass
σ	Bulk density of the small body
R	Radius of the influence sphere dominated by the small body
D	Mean distance from the sun to the small body
U	Gravitational potential
V	Effective potential
H	Hamiltonian function
B	Bézier surface function
G	Gravitational constant
i	Unit of imaginary number
t	Time
τ	Normalized time
κ	Dynamical factor of orbital motion around the small body
ρ	Shape factor of the small body

Ξ	Discriminant index for the stability of the equilibrium point
Θ	Discriminant index for the stability of the periodic orbit
\mathbb{R}	Set of real numbers
\mathbb{N}	Set of natural numbers

Subscripts

A	Asteroid
\odot	The Sun
s	Solar radiation pressure
f	Face of the polyhedron
e	Edge of the polyhedron
e	Reference value in the spherical/ellipsoidal harmonics
FS	Face set of the polyhedron
ES	Edge set of the polyhedron