

Astronomy and Calendars – The Other Chinese Mathematics

Jean-Claude Martzloff

Astronomy and Calendars – The Other Chinese Mathematics

104 BC–AD 1644

Jean-Claude Martzloff
East Asian Civilisations Research Centre
(CRCAO) UMR 8155
The National Center for Scientific Research
(CNRS)
Paris
France

The author is an honorary Director of Research. After the publication of the French version of the present book (2009), he has been awarded in 2010 the Ikuo Hirayama prize by the Académie des Inscriptions et Belles-Lettres for the totality of his work on Chinese mathematics.

ISBN 978-3-662-49717-3 ISBN 978-3-662-49718-0 (eBook)
DOI 10.1007/978-3-662-49718-0

Library of Congress Control Number: 2016939371

Mathematics Subject Classification (2010): 01A-xx, 97M50

© Springer-Verlag Berlin Heidelberg 2016

The work was first published in 2009 by Honoré Champion with the following title: Le calendrier chinois: structure et calculs (104 av. J.C. - 1644).

This work is subject to copyright. All rights are reserved by the Publisher, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilms or in any other physical way, and transmission or information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed.

The use of general descriptive names, registered names, trademarks, service marks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

The publisher, the authors and the editors are safe to assume that the advice and information in this book are believed to be true and accurate at the date of publication. Neither the publisher nor the authors or the editors give a warranty, express or implied, with respect to the material contained herein or for any errors or omissions that may have been made.

Cover illustration: After an illustration from Sun Jianai 孫家鼐 et al. *Qingding Shujing Tushuo* 欽定書經圖說 (*Imperially Commissioned Illustrated Edition of the Classic of History*), first chapter, 1905. This late picture represents the measurement of the Sun's shadow at the summer solstice with a gnomon and its shadow template, in legendary Chinese antiquity. It results from an interpretation of a short passage of the *Shujing* 書經.

Printed on acid-free paper

This Springer imprint is published by Springer Nature
The registered company is Springer-Verlag GmbH Berlin Heidelberg

Omnem movere lapidem
(To leave no stone unturned)
D. Erasmus, *Adagiorum collectanea*,
Paris, 1506-1507, I-4-30

To France

CONTENTS

List of Illustrations	xvii
Acknowledgements	xix
Abbreviations and Conventions	xxiii
Foreword	xxvii
Initial Foreword	xxxi
I Chinese Astronomical Canons and Calendars	1
1 Preliminary Observations	3
The State of the Art	3
Methodological Orientations	7
Computistics and Predictive Astronomy	12
The Paradox of the Chinese Calendar	24
The Calendar and its Calculations	27
The Difficulty of Access to Astronomical Knowledge	28
The Surface and Deep Structures	29
Two Notions of Time	30
The Double History of the Calendar	32
Historical Sources (Surface Structure)	33
Historical sources (Deep Structure)	34
Numbers	35
The Key Ideas of Astronomical Canons	37
Political and Cultural Factors: An Example	41
The Reforms of Astronomical Canons	42
The Bureau of Astronomy	52
The Names of Astronomical Canons	55

2	Description of the Chinese calendar	61
	Limitation and Scope	61
	Fundamental Components	62
	The Day	62
	The Solar Year	62
	The Twenty-Four Solar Breaths	63
	The Seventy-Two Seasonal Indicators	66
	The Five Phases	68
	The Lunar Year	69
	Lunar Months, Ordinary and Intercalary	69
	The Structure of the Lunar Year	70
	The Percentage of Full and Hollow Months	71
	Local Patterns of Full and Hollow Months	72
	The Astronomical Months and the Lunisolar Coupling	73
	The Beginning of the Lunar Year	76
	Dynastic Eras and Concordance Tables	78
	Cycles and Pseudo-Cycles	79
	Definitions	79
	The Denary Cycle	80
	The Duodecimal Cycle	81
	The Inverted Tree	82
	The Sexagenary Cycle	83
	Various Uses of the Sexagenary Cycle	86
	The Nine Color Palaces	87
	The Planetary Week	90
	The Twenty-Eight Mansions	92
	The <i>Jianchu</i> Pseudo-Cycle with Reduplications	94
	The <i>Nayin</i> Cycle with Reduplications	96
	Other Aspects	96
	Festivals and Annual Observances	96
	Irregular Years	99
II	Calculations	105
3	Numbers and Calculations	107
	Modes of Representation of Numbers	107

Various Zeroes	118
The Zero-Circle	125
The History Zero Revisited	128
Numerical Constants	131
The Epoch	135
The Superior Epoch	136
The Support Year	137
The Emerging Year	138
Numbers of Years from the Epoch	138
Changes of Origin	139
Support Days	140
Binomial Representations	141
Fractional Representations	143
Mean and True Elements	144
Definitions	144
Historical Aspects	145
Notation and Terminology	146
Fundamental Elements	147
The Last Solar Breath of a Lunar Year	148
The Numbering of New Moons	148
The Lunisolar Shift	149
Introduction	149
The Epact	149
The intercalary remainder (<i>Runyu</i>)	150
The Monthly Epact and the Intercalary Month	150
Consequences	152
Pathological Calendars	153
4 Mean Elements	157
Mean Elements in Practice	157
Metonic constants	157
Metonic Calculations	158
Justifications	161
Non-Metonic Canons	163
Calculation Variants	166

5 True Elements (618–1280)	169
Introduction	169
True Solar Breaths	170
Some Peculiarities Leading to Simplifications	170
A Technical Term: The <i>ruqi</i>	172
A General Mode of Calculation of the <i>ruqi</i>	173
Another Mode of Calculation	174
The Calculation of the <i>ruqi</i> from Mean Solar Breaths	175
Another Technical Term: The <i>ruli</i>	179
The <i>ruli</i>	179
Tables and Interpolation Techniques	179
Solar Tables	180
Lunar Tables	185
The Solar Correction	186
Further Remarks On the Solar Correction	190
The Lunar Correction	191
Calculations Without Tables	195
6 Later Astronomical Canons	197
The Supremacy of the Inception Granting Canon	197
The Two Last Astronomical Canons	198
Units of Time	200
The Epoch	200
Concordances with Julian Dates	201
The Reform of the Shift Constants	202
Mean Elements	203
Justifications	204
True Lunar Phases	204
True New Moons	204
True Lunar Phases	207
Notes	207
Horary System	213
The Durations of Day and Night	215
The Epoch	216

7	<i>Mo</i> and <i>Mie</i> days	219
	Introduction	219
	Definitions	221
	Immediate Consequences of the Definitions	222
	Calculations Techniques	223
	Justifications	226
	Supplementary Results	232
	Justifications	232
	The Indian Origin of <i>Mo</i> and <i>Mie</i> Days	234
III	Examples of Calculations	239
8	The Quarter-Remainder Canon	241
	Importance	241
	Fundamental Parameters	241
	The Year 119	242
	Initial Calculations	242
	Another Procedure	243
	Other Solar and Lunar Elements	246
	General Structure	254
9	The Luminous Inception Canon	261
	Importance	261
	Fundamental Parameters	261
	The Years 450 and 451	262
	The Years 450 and 451	267
	The Calendar of the Year 450	268
	Guidelines	268
	Translation	269
	Notes	271
	The Two Lunar Eclipses of the Year 451	274
10	The Manifest Enlightenment Canon	277
	Importance	277
	Fundamental Parameters	277
	The Year 877	279

Former Studies	279
The Mean Elements of the Year 877	279
The True Elements of the Year 877	280
True New Moons	289
The Calendar of the Year 877	290
A Printed Almanac of the Year 877	296
General Presentation	296
Some More Details	298
11 The Great Unification Canon	303
Its importance	303
Fundamental Parameters	303
The Year 1417	304
The Intercalary Character of the Year 1417	304
The Mean Elements of the Year 1417	305
The True Moons Phases of the Year 1417	305
The Determination of the Intercalary Month	308
Other Moon Phases	308
Cycles and Pseudo-Cycles	313
Justifications	313
A Calendar for the Year 1417	315
Presentation	315
Monthly Structure	316
Translations	318
12 <i>Mo</i> and <i>Mie</i> Days	325
Preliminary Remarks	325
The <i>Mo</i> days of the year Jiading 11 (1218)	325
The <i>Mie</i> Days of 877	328
The <i>Mo</i> Days of 1417	329
The <i>Mie</i> Days of 1417	330
Afterthoughts	333
Appendices	339
Appendix A The sexagenary cycle	341

Appendix B The Twenty-Four Solar Breaths	343
The Twenty-Four Solar Breaths (104 BC – AD 1644)	344
The Lunisolar Coupling	345
Appendix C The Seventy-Two Seasonal Indicators	346
Appendix D Official Astronomical Canons	350
List of Official Astronomical Canons	351
Metonic Official Astronomical Canons	354
Appendix E Time Constants	357
Appendix F Solar Constants	361
Appendix G Lunar Constants	365
Bibliography	371
Tables of the Chinese Calendar	371
A List of Tables	372
Recent Advances (2012–2014)	382
Computer Programs	382
Primary Sources	385
The Astronomical Canons in the Dynastic Histories	385
Extant Calendars	387
The Most Ancient Extant Calendars	387
Dunhuang Calendars	387
Song Calendars	387
Yuan Calendars	388
Ming Calendars	389
Other Primary Sources	390
Collections of Primary Sources	390
Individual Works	391
Mathematical sources	398
Korean and Japanese sources	399
Japanese sources	400

Rare Sources	401
The Jesuit Reform of Chinese Astronomy	403
Antoine Gaubil	404
Philippe de La Hire	406
Secondary sources	407
COLLECTIVE WORKS	407
BOOKS AND ARTICLES	408
Glossary	441
Index of Names	447
Index of Subjects	453

LIST OF ILLUSTRATIONS

1.1	The lunar eclipse on May 4, 1632	18
1.2	A Chinese planetary ephemeris for the year Jiaqing 10 (1531)	20
1.3	Philippe de la Hire's astronomy without hypotheses . . .	40
2.1	The earliest known representation of the sexagenary cycle	84
3.1	A. Gaubil's full awareness of the centesimal number system	110
3.2	Yearly table of gnomon lengths	113
3.3	Zeroes in a table of the motion of Venus	124
3.4	Types of zeroes in two important mathematical texts from the Song and Yuan dynasties	127
3.5	the cuneiform zero	129
10.1	A part of the ninth month of the printed almanac of the year 877	301
11.1	The division of the months of the calendar for the year 1417 into nine zones	317
11.2	The thirteen first days of the first lunar month of the year 1417	323

ACKNOWLEDGEMENTS

The idea at the origin of this study of the Chinese calendar and its calculations was sparked by the observation that surprisingly little research has been conducted into this domain until now, but it is also the consequence of a much more ancient interest in the history of Chinese mathematics and the mathematical aspects of Chinese astronomy. It would not have been completed without the support of the National Center for Scientific Research (CNRS) and other research centers into sinology and the history of science, and without repeated contacts with professionals of these domains all over the world. The professors Jacques Gernet (honorary professor at the Collège de France) and Jean Dhombres (emeritus director of research at the CNRS and head of study at the École Pratique des Hautes Études en Sciences Sociales (EHESS)) have both actively encouraged its realization and it is a pleasure for me to express my gratitude to them in the first place.

In France, the completion of the *Grand dictionnaire Ricci de la langue chinoise* (*Great Ricci Dictionary of the Chinese Language*) (Paris, Desclée de Brouwer, 2001), in which I have been involved for many years as regards Chinese mathematics, astronomy and the calendar, has been an occasion for me of frequent contacts with members and researchers of the Ricci Institute, notably the late Father Claude Larre S.J. and Élisabeth Rochat de la Vallée. These philological works have then stimulated my interest in all sorts of related issues.

Above all, the CNRS team devoted to research into Chinese civilization, where I have worked with other sinologists, has been essential. More particularly, I have been involved in a collaborative research project on divination and society in Medieval China, notably with my colleagues Marc Kalinowski (Head of Study at the École Pratique des Hautes Études (EPHE)) and Alain Arrault (member of the École Française d'Extrême Orient (EFEO)) from 1999 to 2003. On this occasion, we have been lastingly interested in the description, classification and dat-

ing of about fifty non-official Chinese manuscript calendars from the Dunhuang collections (781–993) held by the Bibliothèque nationale de France and the British Library, mainly. During these years, I had direct exchanges with many Chinese researchers, invited by our research team or met on the occasion of international congresses, notably the Taiwanese professors Ping-yi Chu (Academia Sinica, history and philology), Daiwie Fu (Tsing-Hua University, history department), Yi-long Huang (Tsing-Hua University, history department and later Academician from the Academia Sinica), Wann-Sheng Horng (Taiwan National University mathematics department) and Deng Wenkuan (Research Institute into the Chinese Cultural Heritage, Beijing). All of them have then kindly answered my queries and have kept me informed of ongoing research into the Chinese calendar or related subjects, connected in one way or another with the history of Chinese mathematics.

During the first week of July 2000, I participated in a conference on calendars in general, organized by the historians of the Middle Ages Jacques Le Goff and Perrine Mane and the mathematician Jean Lefort. Then, the multi-faceted aspects of calendrical time in various cultures have usefully highlighted the similarities and the peculiarities of the Chinese case comparatively with so many other possibilities of calendars. Some time before, regular contacts with Tony Lévy (Hebrew mathematics, CNRS), Pierre-Sylvain Filliozat (Sanskritist, Académie des Inscriptions et Belles-Lettres, CNRS, EPHE), André Cauty (Amerindian linguistics, Bordeaux University) and Jim Ritter (Babylonian mathematics, Paris VIII University) have brought to my attention the problems raised by written numerations and zero in general.

In England, I have visited two times the Needham Research Institute in Cambridge, in December 1997 and December 2005. Christopher Cullen (the present Director of this Institute, successor of Professor G.E.R. Lloyd) and the librarian John Moffett, have facilitated my access to its precious documentation and allowed me to work in particularly favorable conditions. I have also met there the historian of astronomy Raymond Mercier (Cambridge University) in Cambridge and elsewhere.

In China, at Beijing, multiples invitations to the Institute of Mathematics and Systems Science (Academia Sinica) and the continued support of Professor Li Wenlin – whose role in the development of the his-

tory of mathematics in contemporary China has been essential – have allowed me to meet, on numerous occasions, Chinese researchers specializing in the history of Chinese mathematics and mathematical astronomy, notably the late Chen Meidong, Liu Dun, Guo Shuchun and Sun Xiaochun. Wang Yusheng too has always been ready to meet efficiently all my requests with an unequaled joviality and receptiveness. In September 1993, I could meet the professors Zhang Peiyu and Li Yong, specialists of Chinese astronomy, at the Nanjing Zijinshan Observatory. At the same time, Professor Xuan Huanan (Nanjing University, Astronomy Department) has kindly forwarded to me recent publications on the subject over many years. At Xi'an and elsewhere, I also have had extensive exchanges with Qu Anjing (Department of Mathematics, Northwest University, Xi'an) and I would like to warmly thank all these researchers and more generally all those who have made my work in China easier.

Lastly, I would like to express my gratitude to the librarians of the Institut des hautes études chinoises, those of the Sorbonne University (Paris) and all the members of the research team devoted to the study of Chinese civilization to which I belong and where I have worked with pleasure and enthusiasm for so many years.

ABBREVIATIONS AND CONVENTIONS

General Abbreviations

- indicates a negative year;
- . As usual, a dot is used to separate the integer and fractional part of a decimal number. Not to be confused with other similar notations, introduced in Chapter 3 of this work and only concerning non-decimal numbers used in Chinese calendrical calculations;
- j.* *juan*, book chapter (sometimes ‘book’), literal meaning :‘roll’, an allusion to antique Chinese books, similar to the *volumina* of Greek and Roman Antiquity.

Bibliographical Notations

- * When a book has been edited several times, an asterisk marks the years of publication of the consulted versions;
- When the publication of some work extends over several years, its initial and final years are separated by a dash;
- / Slash between the various years of edition of a work published several times;

Other Abbreviations

- abbrev. abbreviation
- Ar. Arabic
- astron. ‘astronomy’ or ‘astronomical’
- ca. Latin: circa. Means ‘approximately’
- cal. calendar(s) or calendrical
- Chin. Chinese

denom	denominator
Eng.	English
f.	after a page number means ‘and the following pages’ (also noted ‘ff.’ in other works)
fl.	floruit. Indicates when somebody was active (Latin: flourished).
Heb.	Hebrew
Jap.	Japanese
Lat.	Latin
ms.	manuscript
numer	numerator
proc.	procedure(s)
Skr.	Sanskrit
transl.	translation

Lunar Phases and Special Months

FM	full moon;
FQ	first quarter;
LQ	last quarter;
NM	new moon (also very often noted n_i for some i);
m_{an}	anomalistic month
m_{syn}	synodic month

Reference Works

COLL.	collective work;
COL-astron	<i>Zhongguo kexue jishu dianji tonghui, tianwen juan</i> 中國科學技術典籍通彙, 天文卷 (<i>General Collection of Chinese Scientific and Technical Works, Astronomy</i>), 1993 (full reference p. 390);
COL-math	same reference but for mathematics (see also p. 390);

- DENG-2002 DENG Wenkuan, *Dunhuang Tulufan tianwen lifa yanjiu* 敦煌吐魯番天文曆法研究 (*Research into Calendars and Astronomy from Dunhuang and Turfan*), Lanzhou, Gansu Jiaoyu Chubanshe 甘肅教育出版社, [collection of articles]
- DENG-2010 DENG Wenkuan, 鄧文寬, *Dunhuang tianwen lifa kaosuo* 敦煌天文曆法考索 (*Astronomical Research and Calendar Manuscripts from Dunhuang*) Shanghai, Shanghai Guji Chubanshe 上海古籍出版社, [collection of articles].
- DKW Morohashi Tetsuji 諸橋轍次. *Dai kanwa jiten* 大漢和辭典 (*Great Chinese-Japanese Dictionary*), Tokyo, 1960. References to this dictionary are given as follows: volume-page-item number;
- LIFA Zhang Peiyu 張培瑜, Chen Meidong 陳美東 et al., 2008. *Zhongguo gudai lifa* 中國古代曆法 (*Ancient Chinese Astronomical Canons*), *Zhongguo tianwenxue shi daxi* 中國天文學史大系 (*Great Encyclopedia of Chinese Astronomy*), Beijing, Zhongguo Kexue Jishu Chubanshe, 中國科學技術出版社;
- SIXIANG Chen Meidong 陳美東, 2008. *Zhongguo gudai tianwenxue sixiang* 中國古代天文學思想 (*Ancient Chinese Astronomical Thinking*), *Zhongguo tianwenxue shi daxi* 中國天文學史大系 (*Great Encyclopedia of Chinese Astronomy*), Beijing, Zhongguo kexue Jishu Chubanshe, 中國科學技術出版社;
- WYG *Yingyin Wenyuange siku quanshu* 影印文淵閣四庫全書 (*Reproduction of the Siku quanshu Collection Preserved at the Wenyuange Library*), 1500 vol., 1986 (full reference p. 390);

is pinyin

The Transliteration of Chinese

The transliteration of Chinese used here is the *pinyin* phonetic system, a system adopted in 1958 by the People's Republic of China and now widely accepted all over the world in specialized and non-specialized

publications alike even though the older Wade-Giles system is still rather widespread in the English speaking world.

At the same time, it has been impossible to avoid all sorts of other transcriptions of Chinese names and notions established by custom. But their *pinyin* equivalents or even their original Chinese written forms have also been indicated when necessary.

Chinese Dates

Chinese dates traditionally rely on various and more or less complex formulations. More simply, we have uniformly adopted here the simple format ‘day/lunar month/ lunar year’, where lunar months are denoted by Roman numerals in order to avoid any confusion with Julian or Gregorian dates.

FOREWORD

The study of the ancient Chinese mathematics used for astronomy and the calendar proves that it differs significantly from that of the well-known ‘Nine Chapters’ tradition. It consists in unceasingly reworked procedures devoted to the prediction of celestial phenomena and the calendar. This study reveals unexpected results, notably, *inter alia*, non-decimal number systems and a form of written zero not attested elsewhere, the weight of numerology, the strong link between predictive mathematics and divination and the predominance of empirical observation over theory. These results are of interest not only in the history of Chinese mathematics but also, more generally, in the history of science, including astronomy. Furthermore, a comparison between Chinese and non-Chinese ancient approaches reveals both numerous points of contacts and striking dissimilarities, notably a lasting Chinese belief in the impossibility of long-term mathematical predictions. Lastly, numerous examples of calculations support the general description of mathematical patterns underlying calendrical calculations. The present study is a logical extension of my former *A History of Chinese Mathematics* (Springer, 1996 and 2006) and, as such, is primarily intended for readers interested in the cultural history of Chinese mathematics, with or without any sinological background.

Given the multifactorial nature of the history of mathematics, various subjects, which are an integral part of the history of Chinese mathematics, have also been introduced in the first part of the book, notably the influence of politics on mathematics, the Bureau of astronomy, the secret character of astronomical canons, their names, the importance of numerology and divination. However, these various subjects have always been subordinated to the mathematical aspect of the present study, even though each of them could easily have led to the redaction of a sizeable monograph. By committing to this perspective, I have tried to highlight a number of elements generally not taken into account in available his-

tories of Chinese mathematics, notably a piecewise conception of variable phenomena in terms of phases depending on yin-yang conceptions, tests (or quasi-criteria) delivering only probable results, unusual astronomical tables containing not only lists of predetermined coefficients but also terse procedural instructions ('quasi-tables'), a negative mode of definition – the sole in all extant ancient Chinese mathematics, apparently –, concerning intercalary months. Moreover, beyond punctual peculiarities of this mathematics and, more generally, of Chinese calendrical calculations constituting the core of this work, a broader issue, of interest in the comparative history of science, has also been addressed in order to catch a glimpse of fundamental Chinese conceptions concerning the nature, function and strength of mathematics and the possibility of "laws" of nature. Starting from a corrected and updated version of its former French version, this book has been modified and organized as follows:

- more importance has been granted to methodological problems (Chapter 1);
- the analysis of the fundamental but difficult notion of *li* 曆, respectively meaning 'calendar' in general and 'mathematical astronomy', 'astronomical systems' or 'astronomical canons' in a technical sense, has been significantly developed from a comparative perspective, mainly implying mathematical astronomy from the Islamic and Greek worlds, in the first chapter of this book;
- the analysis of the Chinese belief in the artificial nature of mathematics, and in the impossibility of obtaining immutable predictive techniques based on mathematics, has been reexamined in order to show that, on the contrary, unbounded mathematical predictions were also regarded temporarily as an abstract possibility in the Chinese late medieval and pre-modern context. In the long run, however, the very possibility of obtaining a mathematical formulation of such techniques has continued to be strongly called into question in China;
- the presentation of Chinese mathematical techniques, including those of interpolation and the analysis of solar and lunar tables

has been much more developed. Moreover, the notions of quasi-criteria and quasi-tables have been introduced in order to highlight essential aspects of these other Chinese mathematics;

- a fully worked out example has been added which details the calculation of the dates on which two lunar eclipses occur, recorded in a manuscript calendar for the year 451;
- the bibliographies of primary and secondary sources have been updated in order to take the latest developments (2014) into account;
- last but not least, a large number of further details and new figures have been inserted in various places of the main text. The latter concern, for instance, a Chinese planetary ephemeris, the question of an astronomy without hypotheses, the centesimal system, a schema of an eclipse prediction, the oldest known inscription showing the whole Chinese sexagesimal cycle and, notably, zero.
- a new final section, ‘afterthoughts’, indicates possible directions of interest for future research into the field, concerning, notably, the extension of the present investigation to the mathematical aspects of astronomical canons beyond calendrical calculations, chronological problems and topics concerning more particularly the history of mathematics.

Moreover, like its former version, this book can be used in various ways:

A number of sections can be consulted independently. That is the case for the appendixes and bibliographies, of course. In particular, detailed and updated presentations of almost all available tables of the Chinese calendar, concordance tables and various primary sources have been propounded.

As for the body of the book itself, chapters 1 to 4 are certainly prerequisites but, whereas the first chapter does not involve technical developments and can be read independently, on the contrary, all the notations, definitions and notions introduced in chapters 3 and 4 are constantly used everywhere in the sequel.

More particularly, the fourth chapter grants access to all subsequent developments relying on mean elements which are used in one way or another, exclusively or partly, in all following chapters. In its turn, the fifth chapter introduces true elements (as opposed to mean elements) and related notions intervening in Chinese calendrical calculations. As such it is thus a prerequisite for the examples of calculations developed in the tenth chapter.

The sixth chapter concerns the two latest systems of calendrical calculations (from 1281 to 1644) and the related example of calculations developed at length in the eleventh chapter cannot be read independently.

In the same order of ideas, almost everything contained in the seventh chapter is self-contained, but the examples of related calculations presented in the twelfth chapter also rely on techniques of calculation of mean elements introduced in the fourth chapter.

Lastly, I add that I have entirely composed the French and English versions of the present book from the latest versions of MiKTeX¹ and Texmaker² in order to produce a pdfLaTeX output. Moreover, in order to avoid reencoding all previous files of the French original I had to enter Chinese characters from their Big5 encoding. Consequently, a rare Chinese character, whose *pinyin* transcription is *chong* and which is used in the name ‘Zu Chongzhi’ was not available. However, I could replace it by an homophone, 冫, having a nearly identical graph: it has merely one dot in excess on its left part which should be removed in order to obtain the missing character.³ Moreover, for the same reason, some Japanese words have been reproduced from their ancient forms. However, the correspondence with those now in use can be easily retrieved from current dictionaries.⁴

¹<http://miktex.org/>.

²<http://www.xmlmath.net/texmaker/>.

³The three Chinese characters used in this book for ‘Zu Chongzhi’ are the following: 祖沖之.

⁴For instance: Nelson, A.N., *The Modern Reader’s Japanese-English Character Dictionary*, 2nd Revised Edition, Charles E. Tuttle Company, 12th printing, 1981.

INITIAL FOREWORD

This purpose of this book is to highlight some of the most fundamental mathematical structures underlying the calculation techniques used for the construction of the Chinese historical official calendar, in a way that makes it possible to efficiently determine its main elements over as large a number of years as possible, from a preliminary description of its invariant structure. Apart from technical matters, however, great importance has also been granted to the wider context of this mathematics, particularly its epistemological aspect, which is so important in order to understand its nature, purpose and function.

Unofficial Chinese calendars and non-Chinese calendars currently also used in China, such as the sinicized Muslim calendar, are not included in the present study for the following reasons: we wholly ignore the calculation techniques of the former whereas the latter rely on mathematical techniques utterly different from those of the Chinese calendar.

The historical period retained spans the years of the interval 104 BC–AD 1644, a choice dictated both by the state of manuscript and printed sources handed down to us and by the overall unity of Chinese calendrical calculations developed between these two limits, extending over more than seventeen centuries.

In the case of more ancient years, we do not possess any historical document explaining calendrical calculations while, on the contrary, numerous and detailed treatises are available for most years posterior to 104 BC.

From 104 BC to AD 1644, Chinese calendrical calculation techniques have never ceased to belong to the same family, for they have always been designed in the form of lists of procedures, always formulated and organized in the same way. Overall, calendrical events are mostly dealt with like those of astronomy by only seeking to obtain the best possible precision, irrespective of their unceasingly variable underlying techniques. No less characteristically, the modes of representation

of numbers and the technical terminology they rely on are eminently unstable; they also assign arbitrary patterns to numbers by taking avail of numerological correlations.

By contrast, the period following the year 1644 marks a break with previous traditions: the reform of Chinese astronomy then undertaken successfully by Jesuit astronomers has resulted in a dependency of Chinese calendrical calculations on techniques previously unheard of in China – trigonometry, geometry, logarithms, . . . – used in Renaissance Europe and directly in line with Greek mathematics and astronomy. Hence a discontinuity between former and newer Chinese practices which would doubtlessly deserve a study in its own right. Yet, when certain features of Chinese calendrical calculations from this later period can clarify older practices for one reason or another, we have not ruled out comparisons.

Although, as already noted, official Chinese calendrical and astronomical techniques developed between 104 BC and AD 1644 belong to the same family, its members are extremely numerous and seemingly far apart from each other: in the intervening years, they have been unceasingly reformed no fewer than fifty times. Nothing of the sort exists anywhere else than in the Chinese world.

In order to highlight the main structures organizing this wealthy repository of mathematical techniques, the following approach has been retained:

First, noting that the backbone of the Chinese official calendar has remained identical to itself over time, we have attempted to highlight its invariant structuring principles and ideas.

Second, we have deliberately chosen to formulate in as general a way as possible the abstract techniques underlying Chinese calendrical calculations and the numerical results they lead to. By contrast, their profuse philological and syntactic peculiarities have not been our priority, even though this aspect of the question is certainly also of primary importance for other purposes. Given the inchoative state of this field of study, however, practically everything remains to be done, even when taking into account the always increasing number of specialized publications. Therefore, the need to clarify the general structure of calendrical calculations has seemed more pressing than a study focusing on the

exploration of the quasi-tropical jungle of its specific linguistic manifestations. Nevertheless, such questions have been highlighted when it was obviously desirable to take them into account.

After numerous tentative steps and false trails, we have elaborated a first version of a technique for describing Chinese calendrical calculations from a limited set of ad hoc notions and notations.

Thanks to this tool, it turned out that only a limited number of such techniques exists, modulo a residual number of recalcitrant processes, either seemingly difficult to gain access to because of the incomplete character of Chinese sources or their apparent obscurity, mainly. More precisely, Chinese calendrical techniques fall under two fundamental types: those relying on mean elements and those admitting both mean and true elements, these two notions having their usual astronomical meaning.

On this basis, we have fully described the structural core of all the techniques of the first type and sketched an outline of the second ones, so that it became easier to figure out the overall rationale of the innumerable techniques used to calculate Chinese official calendars from 104 BC–AD 1644.

Hence, also, the conquest of a sort of autonomy enabling us to perform Chinese calendrical calculations in various but equivalent ways, recorded or not in original sources. It follows therefrom that it is not always necessary to follow original procedures to the letter in order to obtain exactly the same results, to grasp their scope, to deduce some of their consequences and to provide answers to, notably, the following questions:

- Is a calculation technique so obtained sufficient in order to restore exactly the content of authentic Chinese official calendars?
- Is it generalizable?

Although the small number of extant authentic calendars issued between 104 BC and AD 1644 precludes the possibility of giving a final and general answer to these questions, it remains that once a calculation method has been formulated, it becomes possible to determine the putative content of Chinese official calendars, even in the case of years for

which authentic calendars are not extant. Hence the possibility to retrieve the theoretical dates of a large number of calendrical events, well beyond the most fundamental ones listed in available tables of the Chinese calendar and concordance tables. But, of course, this is not always possible in all cases if only because not all Chinese calendars have been obtained only from calculations but also from political decisions.

Beyond reconstructed calendrical dates, it is sometimes also possible to deduce completely and globally the general structure of calendars obtained from certain types of procedures, notably those based only on mean elements.

Furthermore, the practice of these calculations shows that the modes of representation of numbers used in calendrical calculation – and more generally in all Chinese astronomical canons too – are unexpectedly not based on decimal representations. Moreover, it also happens that number representations also rely on a particular form of written zero, practically never mentioned by historians of mathematics. It is of course hardly necessary to stress the importance of these two results which are of interest not only for the history of the Chinese calendar and astronomy but also, more broadly, for the history of mathematics since they drastically challenge the usual idea that, over its very long history, China would have been only aware of nothing else than a decimal and positional system of numeration, any other possibility being ruled out.

In order to present these results, the present work has been divided into three parts:

The first part expounds the principles on which this study is based. Then a presentation of the history of the Chinese calendar follows, both from the perspectives of its specific content and calculations, together with related questions of interest, such as a list of all irregular years of the Chinese calendar.

The second part focuses on the fundamental technical, mathematical and astronomical aspects essential to any description of Chinese calendrical calculations: the representation of numbers and numerical constants, astronomical and non-astronomical, the question of the determination of the origin of time and other technical notions peculiar to lunisolar calendars, such as the lunisolar shift, or epact, and the rule for determining intercalary months. On this basis, the details of par-

ticular techniques of interest in the calculation of the Chinese calendar are tackled by distinguishing those using either mean elements, true elements or both sorts of elements. Then, two little known elements of the Chinese calendar, namely the *Mo* 沒 and *Mie* 滅 days, are introduced with a wealth of details because they tend to prove the influence of Indian culture on the Chinese calendar.

The third and last part contains some examples of fully developed calculations for calendars of given years, their results being compared, when possible, with the content of authentic calendars and when not, with the calendrical data listed in chronological tables of the Chinese calendar. In addition, a large number of examples of specific calculations are provided everywhere in the main text.

Thereafter, a series of appendixes also provides systematic tables giving lists of numerical constants indispensable when performing calendrical calculations, together with a chronological list of Chinese official astronomical canons.

Lastly, the bibliography of primary sources presents the most important references concerning both the study of the Chinese calendar, its calculations and Chinese chronology. In its turn, the bibliography of secondary sources contains an extensive list of publications in Chinese, Japanese and Western languages.

Chinese calendrical calculations in general constitute a vast domain. However, they represent only a minute fraction of the wider domain of Chinese mathematical astronomy. So far, however, this astronomy has often been regarded as reducible to a purely qualitative science, based on an accumulation of precise observations and no mathematics, contrary to the other great astronomies from Antiquity and the Middle Ages.

If, by stressing the role of mathematics with respect to the limited domain of Chinese calendrical calculations, the present work can encourage historians of science and sinologists to take a greater account of the eminently mathematical character of traditional Chinese astronomy, our objective would have been achieved.