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Fundamentals of Spherical Array Processing

 Springer

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To my parents, Nitzan and Rivka Rafaely

Preface

Microphone arrays and associated array processing techniques have been developed for a wide range of applications over the past few decades. These applications include speech communication, music recording, room acoustics analysis, noise control and acoustic holography, defense and security, entertainment, and many more. In the cases of speech in rooms and music in concert halls, the sound tends to travel throughout the entire enclosed space, producing a three-dimensional sound field. Microphone arrays that effectively measure and process three-dimensional sound fields typically require the positioning of microphones within a volume in three-dimensional space. Planar arrays, mounted on an enclosure wall, have been studied for several decades, while more recently, spherical arrays, in which microphones are mounted around a rigid sphere, for example, have been developed. These offer several advantages over classical linear, rectangular, or circular arrays:

- (i) The sphere, having complete rotational symmetry, facilitates spatial filtering, or beamforming, that can be designed to effectively enhance or attenuate sources in any direction.
- (ii) Array processing and performance analysis can be formulated in the spherical harmonics domain, which is the Fourier domain for the sphere. This domain facilitates efficient algorithms and extensive acoustic modeling of both the array and the surrounding sound field.
- (iii) Beamforming can be efficiently implemented by decoupling beam pattern design from beam pattern steering, therefore providing simplicity and flexibility in array realization.

These advantages have motivated an increasing number of researchers in recent years to develop spherical microphone array systems, to study spherical array configurations, to develop algorithms for spherical arrays, and to apply these arrays in a wide range of applications. This growing activity has provided the author with the motivation and inspiration to write this book, with the aim of presenting the fundamentals of spherical array processing in a tutorial manner suitable for researchers, graduate students, and engineers interested in this topic.

The first two chapters provide the reader with the necessary mathematical and physical background, including an introduction to the spherical Fourier transform and to the formulation of plane-wave sound fields in the spherical harmonics domain. The third chapter covers the theory of spatial sampling, which becomes useful when selecting the positions of microphones to sample sound pressure functions in space. The next chapter presents various spherical array configurations, including the popular configuration based on a rigid sphere. The fifth chapter introduces the concept of beamforming and its basic equations, including popular design methods such as delay-and-sum and regular beamforming. The following chapter presents methods for the optimal design of beam patterns, formulated to achieve various objectives, such as maximum robustness, maximum directivity, or minimum side-lobe level. The final chapter develops more advanced array processing algorithms, such as the minimum variance distortionless response (MVDR) algorithm. These algorithms aim to enhance a desired signal while attenuating undesired noise components in the sound field by exploring their unique formulation in the spherical harmonics domain.

My own interest in spherical array processing began during a six-month visit to the sensory communication group at MIT in 2002, working with Julie Greenberg and greatly enjoying the stimulating vibe of Boston. I would like to thank Julie for providing this opportunity, for the hospitality, and for the helpful discussions. During my visit to Boston I was exposed to the inspiring publications on spherical arrays by Jens Meyer and Gary Elko. Their pioneering work planted the seeds that later flourished to an extensive research effort at my lab, the acoustics laboratory, Ben-Gurion University of the Negev. The research at the acoustics laboratory was pursued by an invaluable cooperation with a great number of research students, postdoctoral researchers, and visitors. The relaxed atmosphere at the lab, the great teamwork, and the endless discussions were the fuel that kept the writing of this book viable. I would like to express great thanks to the acoustics laboratory researchers: Dr. Jonathan Sheaffer, Dr. Jonathan Rathsam, Dr. Noam Shabtai, Dr. Dror Lederman, Dr. Yotam Peled, Dr. Etan Fisher, Vladimir Tournabin, Hai Morgenstern, David Alon, Koby Alhailany, Mickey Joffet, Elad Cohen, Dima Lvov, Or Nadiri, Shahar Villeval, Tal Szpruch, Nejem Huliheh, Ilan Ben-Hagai, Tomer Peleg, Amir Avni, Morag Agmon, Maor Klieder, Dima Haykin, Itai Peer, and Ilya Balmages. Also, special thanks to Dr. Franz Zotter for the helpful comments on a draft version of the manuscript made during a visit to the lab. Thanks also to Debbie Kedar for the prompt and professional editing and proofreading of this book. Finally, thanks to my family, Vered, Asaf, Yonathan, and Tal, for providing love therapy that time and again pulled me out of the writing stumbles and falls.

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