Undergraduate Texts in Mathematics

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Algebra

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The cover design is based on John Croker's medal of Newton. The figure is that of winged Science holding a tablet upon which appears the solar system. More about it in D. E. Smith, *History of Mathematics*, Vol. I, (Dover).

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Preface

There is no one best way for an undergraduate student to learn elementary algebra. Some kinds of presentations will please some learners and will disenchant others. This text presents elementary algebra organized according to some principles of universal algebra. Many students find such a presentation of algebra appealing and easier to comprehend. The approach emphasizes the similarities and common concepts of the many algebraic structures. Such an approach to learning algebra must necessarily have its formal aspects, but we have tried in this presentation not to make abstraction a goal in itself. We have made great efforts to render the algebraic concepts intuitive and understandable. We have not hesitated to deviate from the form of the text when we feel it advisable for the learner. Often the presentations are concrete and may be regarded by some as out of fashion. How to present a particular topic is a subjective one dictated by the author's estimation of what the student can best handle at this level. We do strive for consistent unifying terminology and notation. This means abandoning terms peculiar to one branch of algebra when there is available a more general term applicable to all of algebra. We hope that this text is readable by the student as well as the instructor. It is a goal of ours to free the instructor for more creative endeavors than reading the text to the students.

We would have preferred to call this book *College Algebra* because this was the name of the standard algebra course for undergraduate students in the United States for many years. Unfortunately, the name "College Algebra" now seems firmly attached to a body of material taught in the 1930's. Perhaps in time the name "College Algebra" will once again describe the algebra studied by college students. Meanwhile we have names like "Modern Algebra" and "Abstract Algebra" using inappropriate modifiers.

Included in the first half of the text and providing a secondary theme are a development and construction of the number systems: natural numbers (Sections 3.1–3.4), integers (Sections 3.5–3.6), fractions or rational numbers (Section 4.5), and complex numbers (Section 5.8). The construction of the real number system is properly a topic in analysis; we refer the reader to reference [10, p. 234] for an algebraically oriented presentation. The use of the integers as exponents and multiples as in secondary school algebra is covered in detail (Section 4.4). All of the material on number systems can be stressed advantageously by those students preparing for school teaching.

As in all texts, size considerations eventually begin to exercise influence. Group theory is not stressed in this text although there is a respectable amount of material for an elementary text in Chapter 9. There is no Galois theory in this text. Although lattice theory is a central concept for universal algebra we have pretty well omitted study of that area. For that reason and others, this cannot be considered to be an elementary text in universal algebra.

Considerable attention has been paid to the algebraic properties of functions and spaces of functions. One of the primary uses of algebra for an undergraduate is in his analysis courses. We hope that the attention we have paid to functions will be found rewarding by the student in his analysis courses and in turn we hope that the somewhat concrete nature of spaces of functions helps illuminate some of the algebraic structures by being tangible examples of those structures.

Chapters 1–5 are devoted to rings, Chapters 6, 7, and 10 to linear algebra, Chapter 9 to monoids and groups, and Chapter 8 to algebraic systems in general. We envision the text being used for a year's course in algebra, for a one semester course not including linear algebra, or for a linear algebra course. A shorter course in algebra might consist of Chapters 1–5, omitting possibly Section 3.8, Section 4.6, and parts of Chapter 5, supplemented by Sections 9.1–9.4. A course in linear algebra for students already familiar with some of the topics included in the first five chapters could concentrate on Chapters 6, 7, and 10 after reviewing Sections 5.1–5.6. Ideally we envision the book for a one-year course covering all the chapters.

The questions at the end of each section are to help the reader test his reading of the section. Certainly the section ought be read carefully before attempting to answer the questions. Many of the questions are tricky and hang upon small points; more than one of the answers may be correct. The exercises provide for practice and gaining a better knowledge of the material of the section. It is our practice to use in examples and in the exercises some material on an intuitive bases before the material is treated in the text more formally. Provided one guards against circular reasoning this provides for a more immediate illustration of the principles the student is trying to understand.

Algebra as an undergraduate course is frequently the subject in which a student learns a more formal structure of definitions, theorems, and proofs.

The elementary calculus is often a more intuitively presented course and it is left to the algebra course to institute the more formal approach to mathematics. For this reason the student should be very aware of what are definitions, what are theorems, and what is the difference between them.

We now make several comments on style. In this text sentences are frequently begun with symbols which may happen to be small letters. e is a transcendental number. We consider such symbols proper nouns and beg forgiveness; we have found the practice of avoiding such sentences too limiting. Secondly, we use a number of run-on sentences connected with *if and only if*. They are too logically appealing to avoid. We leave to the reader without comment all other perversions of the queen's English.

It is our opinion that one of the most rewarding things a student of mathematics can learn is some of the history of the subject. Through such knowledge it is possible to gain some appreciation of the growth of the subject, its extent, and the relationships between its various parts. We cannot think of any other area where such a little effort will reap such a bountiful harvest. For reasons of length and cost we have not included historical material in this text despite the opinion just expressed. We recommend the purchase, reading, and rereading of at least one mathematical history book: see the references for suggestions.

The author wishes to thank Bucknell University for a sabbatical leave to work on the manuscript for this book, the editors of Springer-Verlag for their encouragement and advice, and several readers for their suggestions. All errors I claim for myself.

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