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Continuous Martingales and Brownian Motion

With 8 Figures

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Preface

This book focuses on the probabilistic theory of Brownian motion. This is a good topic to center a discussion around because Brownian motion is in the intersection of many fundamental classes of processes. It is a continuous martingale, a Gaussian process, a Markov process or more specifically a process with independent increments; it can actually be defined, up to simple transformations, as *the* real-valued, centered process with independent increments and continuous paths. It is therefore no surprise that a vast array of techniques may be successfully applied to its study and we, consequently, chose to organize the book in the following way.

After a first chapter where Brownian motion is introduced, each of the following ones is devoted to a new technique or notion and to some of its applications to Brownian motion. Among these techniques, two are of paramount importance: stochastic calculus, the use of which pervades the whole book and the powerful excursion theory, both of which are introduced in a self-contained fashion and with a minimum of apparatus. They have made much easier the proofs of many results found in the epoch-making book of Itô and McKean: Diffusion Processes and their Sample Paths, Springer (1965).

Furthermore, rather than working towards abstract generality, we have tried to study precisely some important examples and to carry through the computations of the laws of various functionals or random variables. Thus we hope to facilitate the task of the beginner in an area of probability theory which is rapidly evolving. The later chapters of the book however, will hopefully be of interest to the advanced reader.

We strove to offer, at the end of each section, a large selection of exercises, the more challenging being marked with the sign * or even **. On one hand, they should enable the reader to improve his understanding of the notions introduced in the text. On the other hand, they deal with many results without which the text might seem a bit "dry" or incomplete; their inclusion in the text however would have increased forbiddingly the size of the book and deprived the reader of the pleasure of working things out by himself. As it is, the text is written with the assumption that the reader will try a good proportion of them, especially those marked with the sign #, and in a few proofs we even indulged in using the results of foregoing exercises.

The text is practically self-contained but for a few results of measure theory. Besides classical calculus, we only ask the reader to have a good knowledge of basic notions of integration and probability theory such as almost-sure and in the mean convergences, conditional expectations, independence and the like. Chapter 0 contains a few complements on these topics. Moreover the early chapters include some classical material on which the beginner can hone his skills.

Each chapter ends up with notes and comments where, in particular, references and credits are given. In view of the enormous literature which has been devoted to Brownian motion and related topics, we have in no way tried to draw a historical picture of the subject and apologize in advance to those who may feel slighted. Likewise our bibliography is not even remotely complete and leaves out the many papers which deal with the relationships of Brownian motion with other fields of Mathematics such as Potential Theory, Harmonic Analysis, Partial Differential Equations and Geometry. A number of excellent books have been written on these subjects some of which we discuss in the notes and comments.

Finally, it is a pleasure to thank those who have offered useful comments on the first drafts in particular J. Jacod, P.A. Meyer, B. Maisonneuve and J. Pitman. Our special thanks go to J.F. Le Gall who put us straight on an inordinate number of points and Shi Zhan who has helped us with the exercises. Each chapter of this book has been taught a number of times by the authors in the last decade, either in a "Cours de 3° Cycle" in Paris or in "crash courses" on Brownian motion; we would like to seize this opportunity of thanking our audiences for their warm response. Last but not least, Josette Saman a pris une part essentielle dans la préparation matérielle du manuscrit et nous l'en remercions bien vivement.

Paris, October 1990

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