# Part II. CAT($\kappa$) Spaces

In Part I we assembled basic facts about geometric notions such as length, angle, geodesic etc., and presented various constructions of geodesic metric spaces. The most important of the examples which we considered are the model spaces $M_\kappa^n$. The central role which these spaces play in the scheme of this book was explained in the introduction: we seek to elucidate the structure of metric spaces by comparing them to $M_\kappa^n$; if favourable comparisons can be drawn then one can deduce much about the structure of the spaces at hand. We are now in a position to set about this task.

In Part II we shall study the basic properties of spaces whose curvature is bounded from above by a real number $\kappa$. Roughly speaking, a space has curvature $\leq \kappa$ if every point of the space has a neighbourhood in which geodesic triangles are no fatter than their comparison triangles in $M_\kappa^2$. In Chapter 1 we give several precise formulations of this idea (all due to A.D. Alexandrov) and prove that they are equivalent. In subsequent chapters we develop the theory of spaces which satisfy these conditions, concentrating mainly (but not exclusively) on the case of non-positive curvature. We punctuate our discussion of the general theory with chapters devoted to various classes of examples.

*Unless further qualification is made, in all that follows $\kappa$ will denote an arbitrary real number. The diameter of $M_\kappa^2$ will be denoted $D_\kappa$ (thus $D_\kappa$ is equal to $\pi/\sqrt{\kappa}$ if $\kappa > 0$, and $\infty$ otherwise).*