

Uwe Mackenroth

Robust Control Systems

Springer-Verlag Berlin Heidelberg GmbH

Uwe Mackenroth

Robust Control Systems

Theory and Case Studies

With 221 Figures



Springer

Professor Dr. Uwe Mackenroth
Fachhochschule Lübeck
University of Applied Sciences
FB Maschinenbau und Wirtschaftsingenieurwesen
Mönkhofer Weg 136-140
23562 Lübeck
Germany

ISBN 978-3-642-05891-2 ISBN 978-3-662-09775-5 (eBook)
DOI 10.1007/978-3-662-09775-5

Library of Congress Control Number: 2004102326

This work is subject to copyright. All rights are reserved, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitations, broadcasting, reproduction on microfilm or in any other way, and storage in data banks. Duplication of this publication or parts thereof is permitted only under the provisions of the German copyright Law of September 9, 1965, in its current version, and permission for use must always be obtained from Springer-Verlag. Violations are liable for prosecution under the German Copyright Law.

springeronline.com

© Springer-Verlag Berlin Heidelberg 2004

Originally published by Springer-Verlag Berlin Heidelberg New York in 2004.
Softcover reprint of the hardcover 1st edition 2004

The use of general descriptive names, registered names trademarks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

Typesetting: camera-ready by author
Cover design: medio Technologies AG, Berlin
Printed on acid free paper 62/3020/M - 5 4 3 2 1 0

*For my family:
Gabriela, Gisela and Julia*

Preface

Control engineering is an exciting and challenging field within the engineering sciences, because by its very nature it is a multidisciplinary subject and because it plays a critical role in many practical technical applications. To be more specific: control theory brings together such different fields as electrical, mechanical and chemical engineering and applied mathematics. It plays a major role in engineering applications of all kinds of complexity. This may be a dc motor, a robot arm, an aircraft, a satellite, a power plant or a plant for the chemical industries such as a distillation column. On the other hand, the developments in the last two or three decades have shown that control engineering requires very solid skills in applied mathematics and therefore has become highly attractive for applied mathematicians, too.

The most fundamental idea of control theory is to change the dynamical behavior of a technical system (the “plant”) by a device called the “controller” such that the dynamics gets certain desired properties. In almost all cases, the controller design is done by building a mathematical model, which can be emulated on a computer and serves as the basis for controller synthesis. It is obvious that the model represents the real system only up to a certain degree of accuracy. Despite this, the controller has to work for the real system as well as for the model; in other words, it has to be “robust” against errors in the mathematical model. Robustness has always played a key role in control theory, right from the beginning, but a considerable amount of theoretical research was necessary to find analysis and synthesis tools which work satisfactorily also for complex plants. In particular, this holds for plants with several inputs and outputs.

The intention of this textbook is to give a self-contained introduction to control theory which emphasizes the modern aspects concerning the design of controllers having prescribed performance and robustness properties. No prior knowledge about control theory is required; the most important facts from classical control are concisely presented in Chaps. 2–4. They are formulated in the light of the modern theory in order to get an early understanding of its most important ideas. The book is written at a graduate level for students of electrical or mechanical engineering sciences or of applied mathematics and may also serve as a reference for control practitioners.

During the last two decades, control theory has changed its character: although

it is a well-established branch of the engineering sciences, it may also be viewed as a discipline of applied mathematics. This is a point which deserves some attention. Almost all control theory methods require some mathematics, and this is also true for the methods based on the Nyquist stability criterion presented in every undergraduate course, but for these the mathematical reasoning can be kept at a moderate level. Even for a complex plant, the role of the controller parameters is well understood and controller design can successfully be done step by step by an engineer who is experienced and has a good intuition.

For modern methods which use \mathcal{H}_∞ optimization or the structured singular value μ the situation is more involved. Probably most people who see the conditions characterizing \mathcal{H}_∞ suboptimal controllers for the first time will find them messy and hard to understand. In fact, there is no way to make these conditions plausible by intuitive engineering reasoning. The only way to accept them as natural and reasonable is to study the complete rigorous mathematical proofs. On the other hand, the application of the more advanced methods, where the weights for performance and the uncertainty structure are specified, requires again a solid engineering experience and intuition.

For the preparation of a textbook dealing with an advanced presentation of control theory, these considerations have some strong implications. In particular, there are only two possibilities concerning the proofs, namely to skip them all or to present them in full detail. Skipping them all would be an admissible way for students or practitioners who are only interested in applications, since the use of the methods, which means mainly to apply the commercial software, does not require a very detailed knowledge of the theory standing behind it. On the other hand, for readers who wish to get a deeper understanding of modern control theory, this possibility is unsatisfactory, and for this reason I have decided to present the modern theory completely as far as it is needed. A study of the theory requires a basic knowledge of linear algebra and analysis and in particular some familiarity with the mathematical way of thinking.

In the introduction it is shown how both kinds of readers can gain benefit from this book. In this context it must be emphasized that a large portion of the text is devoted to elementary examples and advanced case studies. The elementary examples answer questions like: How can a \mathcal{H}_∞ or a μ controller for a simple second order SISO plant without or with uncertainty be compared with a conventional PID controller? The case studies are carried out in full detail and show how the modern methods can be applied to advanced problems. They make intensive use of MATLAB, in particular of the Control Systems Toolbox and the μ -Analysis and Synthesis Toolbox.

The main reason for writing this book was the following. There are many excellent textbooks which present control theory based on the Nyquist criterion and on elementary state space methods. On the other side, there are a lot of highly specialized books which report on recent developments described in the original literature. Books which present the modern theory in detail without being a research monograph or a research-oriented book are comparatively rare. It is in this

sense that the present book should be understood. Its philosophy is to treat advanced theory only so far as commercial software is available in which the new algorithms are programmed.

The most fundamental theoretical parts of the book deal with \mathcal{H}_∞ synthesis using two Riccati equations and robustness based on the structured singular value μ . They are written in the spirit of the famous monograph of Zhou, Doyle, and Glover [112]. A completely different approach to \mathcal{H}_∞ synthesis is based on linear matrix inequalities (LMIs) and will also be presented here. A book treating \mathcal{H}_∞ theory and robustness by LMIs in much greater detail is Dullerud and Paganini [37]. An (incomplete) list of further important books on advanced control theory in the sense discussed above, but with somewhat different intentions to that of this book, is Burl [17], Sanchez-Pena and Sznajder [80], Skogestad and Postlethwaite [92], and Trentelman, Stoorvogel, and Hautus [97].

Uwe Mackenroth
September 2003

Contents

1 Introduction	1
1.1 Control Systems: Basic Definitions and Concepts	1
1.2 Outline of the Book	8
1.3 The Main Steps of Controller Design	11
2 Rational Transfer Functions	17
2.1 Introductory Examples	17
2.1.1 Plants for Pressure Control.	17
2.1.2 Electromechanical Plants	20
2.2 Basic Properties	23
2.2.1 Definitions and Product Decomposition	23
2.2.2 Stability	28
2.2.3 Frequency Response	32
2.3 Elementary Transfer Functions	34
2.3.1 Type-0 Systems of First or Second Order	34
2.3.2 Further Basic Systems	38
3 SISO Feedback Systems	41
3.1 The Basic Transfer Functions	42
3.2 Internal Stability	44
3.3 Stationary Behavior of the Feedback Loop	48
3.4 Nyquist Stability Criterion	50
3.5 Requirements for Feedback Systems	54
3.5.1 Performance	54
3.5.2 Robust Stability	56
3.5.3 Robust Performance	60
4 Classical Design Methods	63
4.1 Specification of the Closed-Loop System	63
4.1.1 Robust Stability and Phase Margin	63
4.1.2 Performance and Gain Crossover Frequency	65

4.2 Controller Design with the Nyquist Criterion	69
4.2.1 Design Method	69
4.2.2 Case Study: Controller Synthesis for a DC Motor	74
4.3 Generalized Structure of SISO Feedback Systems	78
4.4 Summary and Outlook	83
Notes and References	84
5 Linear Dynamical Systems	85
5.1 Linearization	86
5.2 General Properties of Linear Systems	88
5.2.1 Solution Formula and Transfer Matrices	88
5.2.2 Stability	92
5.2.3 Frequency Response	96
5.3 Controllability	97
5.3.1 Realization of Transfer Functions and Controller-Canonical Form.	97
5.3.2 Conditions for Controllability.	102
5.4 Observability	108
5.5 State Space Realizations for Transfer Matrices	113
5.6 Poles and Zeros of Multivariable Systems	118
5.7 Lyapunov Equations	122
5.8 Linear Dynamical Systems with Stochastic Inputs	125
5.8.1 Gaussian Random Processes	125
5.8.2 Solutions of Linear Differential Equations with White Noise Inputs	127
5.8.3 Colored Noise and Shaping Filters	131
Notes and References	132
6 Basic Properties of Multivariable Feedback Systems	133
6.1 Controllers with State Feedback	134
6.2 Estimation of the State Vector	139
6.2.1 Luenberger Observer and Observer-Based Controllers	139
6.2.2 Reduced Observer	141
6.3 Pole Placement for SISO Systems	144
6.3.1 Formula for the Feedback Gains	144
6.3.2 Robust Pole Placement	146
6.3.3 Example: Pole Placement for a Second-Order System	152
6.4 Pole Placement for MIMO Systems.	155
6.4.1 Pole Placement and Controllability	155
6.4.2 Robust Eigenstructure Assignment	157
6.5 General Feedback Systems	160
6.5.1 Structure of General Feedback Systems.	160
6.5.2 Internal Stability and Generalized Nyquist Criterion	165
Notes and References	170

7 Norms of Systems and Performance	171
7.1 Norms of Vectors and Matrices	172
7.2 Norms of Systems	175
7.2.1 The Hardy-Space \mathcal{H}_2 and the Laplace Transform	175
7.2.2 The \mathcal{H}_2 Norm as an Operator Norm	178
7.2.3 The Hardy Space \mathcal{H}_∞ and an Induced Operator Norm	180
7.3 Calculation of Operator Norms	183
7.4 Specification for Feedback Systems	187
7.5 Performance Limitations	190
7.6 Coprime Factorization and Inner Functions	195
Notes and References	199
8 \mathcal{H}_2 Optimal Control	201
8.1 LQ Controllers	202
8.1.1 Controller Design by Minimization of a Cost Functional	202
8.1.2 Algebraic Riccati Equation	207
8.2 Characterization of \mathcal{H}_2 Optimal Controllers	214
8.2.1 Problem Formulation and Characterization Theorem	214
8.2.2 State Feedback	217
8.2.3 Proof of the Characterization Theorem	220
8.3 Kalman Bucy Filter	224
8.3.1 Kalman Bucy Filter as Special \mathcal{H}_2 State Estimator	224
8.3.2 Kalman Filtering with Non-White Noise	227
8.3.3 Range and Velocity Estimate of an Aircraft	229
8.4 LQG Controller and further Properties of LQ Controllers	231
8.4.1 LQG Controller as Special \mathcal{H}_2 Controller	231
8.4.2 An Estimate for the Input Return Difference Matrix	233
8.4.3 Poles of LQ Feedback Systems and Loop Transfer Recovery	235
8.5 Related Problems	238
8.5.1 Optimal Control Problems	238
8.5.2 Systems Described by Partial Differential Equations	242
Notes and References	247
9 \mathcal{H}_∞ Optimal Control: Riccati-Approach	249
9.1 Formulation of the General \mathcal{H}_∞ Problem	250
9.2 Characterization of \mathcal{H}_∞ Suboptimal Controllers by Means of Riccati Equations	252
9.2.1 Characterization Theorem for Output Feedback	252
9.2.2 Outline of the Proof	254
9.2.3 Contraction and Stability.	258
9.3 \mathcal{H}_∞ Control with Full Information	261
9.3.1 Mixed Hankel-Toeplitz Operators	261
9.3.2 Proof of the Characterization Theorem for Full Information	267
9.4 Proof of the Characterization Theorem for Output Feedback	271

9.5 General \mathcal{H}_∞ Problem: Scaling and Loop Shifting	276
9.6 Mixed Sensitivity Design	281
9.6.1 Weighting Schemes	281
9.6.2 Pole-Zero Cancellations	284
Notes and References	288
10 \mathcal{H}_∞ Optimal Control: LMI-Approach and Applications	289
10.1 Characterization of \mathcal{H}_∞ Suboptimal Controllers by Linear Matrix Inequalities.	290
10.1.1 Bounded Real Lemma	290
10.1.2 Nonconvex Characterization of \mathcal{H}_∞ Suboptimal Controllers	292
10.1.3 Convex Characterization of \mathcal{H}_∞ Suboptimal Controllers . .	298
10.2 Properties of \mathcal{H}_∞ Suboptimal Controllers	303
10.2.1 Connection between Riccati- and LMI-Approaches	303
10.2.2 Limiting Behaviour	304
10.3 \mathcal{H}_∞ Synthesis with Pole Placements Constraints	305
10.3.1 LMI Regions	305
10.3.2 Design of \mathcal{D} -stable \mathcal{H}_∞ Controllers	307
10.4 Gain Scheduling	310
10.5 \mathcal{H}_∞ Optimal Control for a Second Order Plant	315
10.5.1 Choice of the Weights	315
10.5.2 Plant with Nonvanishing Damping.	316
10.5.3 Plant with a Pole Pair on the Imaginary Axis	321
Notes and References	326
11 Case Studies for \mathcal{H}_2 and \mathcal{H}_∞ Optimal Control	327
11.1 Control of an Inverted Pendulum	328
11.1.1 Analysis of the Plant Dynamics	328
11.1.2 Design of an LQ Controller	332
11.1.3 Design of an LQG Controller	336
11.2 Control of a Continuously Stirred Tank	339
11.2.1 Analysis of the Plant Dynamics	339
11.2.2 Controller Design by Pole Positioning.	342
11.3 Control of Aircraft	345
11.3.1 Nonlinear and Linearized Dynamics.	345
11.3.2 Analysis of the Linearized Plant Dynamics	351
11.3.3 Improvement of the Lateral Dynamics by State Feedback. .	355
11.3.4 LQ Controller for the Lateral Motion	358
11.3.5 \mathcal{H}_∞ Controller for the Lateral Motion	362
12 Representation of Uncertainty	367
12.1 Model Uncertainty	367
12.2 Unstructured Uncertainties	369
12.3 Structured Model Uncertainties	373

12.3.1	Introductory Examples	373
12.3.2	Structured State Space Uncertainties	375
12.3.3	Parameter Uncertainty for Transfer Functions	379
12.4	General Framework for Uncertainty	380
12.5	Example	383
13	Synthesis of Robust Controllers	387
13.1	Small Gain Theorem	388
13.2	Robust Stability Under Stable Unstructured Uncertainties	392
13.3	Structured Singular Value μ	397
13.3.1	Basic Idea and Definition	397
13.3.2	Basic Properties of the Structured Singular Value μ	398
13.3.3	Estimates for μ	401
13.4	Structured Robust Stability and Performance	404
13.4.1	Robust Stability	404
13.4.2	Robust Performance	406
13.5	D-K Iteration	410
13.6	Reduction of the Controller Order	414
13.6.1	Balanced Realizations	414
13.6.2	Balanced Truncation	416
13.7	Robust Control of a Second-Order Plant	420
	Notes and References	427
14	Case Studies for Robust Control	429
14.1	Robust Control of Aircraft	430
14.1.1	\mathcal{H}_∞ Controller for the Lateral Control with Good Disturbance Rejection	430
14.1.2	Weightings for the Robust Controller	432
14.1.3	D-K Iteration and Simulation Results	435
14.1.4	Comparison with the LQ Controller	439
14.2	Robust Rudder Roll Stabilization for Ships	441
14.2.1	Nonlinear and Linear Plant Model	441
14.2.2	Separate Design for the Controllers for Course and Roll Motion	445
14.2.3	Robust Control of Course and Roll Motion	450
14.3	Robust Control of a Guided Missile.	454
14.3.1	Linear Plant Model	454
14.3.2	Design of a Robust Controller	459
14.3.3	Control of the Elastic Missile	465
14.4	Robust Control of a Distillation Column	467
14.4.1	Nonlinear Model of the Column	467
14.4.2	Nonlinear Model in the Case of Perfect Level Control	471
14.4.3	Linear Plant Model and PI Controller	474
14.4.4	Performance Specification and Robustness Analysis	

of the PI Controller	478
14.4.5 μ Optimal Controller.	481
Notes and References	485
A Mathematical Background	487
A.1. Linear Algebra	487
A.1.1 Linear Mappings and Matrices	487
A.1.2 Eigenvalues and Eigenvectors	489
A.1.3 Self-Adjoint, Unitary and Positive Definite Matrices	490
A.1.4 Matrix Inversion and Determinant Formulas	494
A.1.5 A Minimum Norm Problem for Matrices.	495
A.2. Analysis	497
A.2.1 Banach and Hilbert Spaces	497
A.2.2 Operators	500
A.2.3 Complex Analysis	502
Notes and References	503
Bibliography	505
Notation and Symbols	513
Index	515