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D. Boccaletti   G. Pucacco

Theory of Orbits

Volume 2:
Perturbative and Geometrical Methods

With 81 Figures

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Dino Boccaletti
Università degli Studi di Roma "La Sapienza"
Dipartimento di Matematica "Guido Castelnuovo"
Piazzale Aldo Moro, 2
00185 Roma, Italy
e-mail: boccaletti@uniroma1.it

Giuseppe Pucacco
Università degli Studi di Roma "Tor Vergata"
Dipartimento di Fisica
Via della Ricerca Scientifica, 1
00133 Roma, Italy
e-mail: Giuseppe.Pucacco@roma2.infn.it

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To my wife Maria Grazia
for her love and understanding.
And to the memory of my father,
Aldo Boccaletti,
my first teacher

To Marinella
for her warmth and love

Preface

Half a century ago, S. Chandrasekhar wrote these words in the preface to his celebrated and successful book:[1]

> In this monograph an attempt has been made to present the theory of stellar dynamics as a branch of classical dynamics – a discipline in the same general category as celestial mechanics. [...] Indeed, several of the problems of modern stellar dynamical theory are so severely classical that it is difficult to believe that they are not already discussed, for example, in Jacobi's Vorlesungen.

Since then, stellar dynamics has developed in several directions and at various levels, basically three viewpoints remaining from which to look at the problems encountered in the interpretation of the phenomenology. Roughly speaking, we can say that a stellar system (cluster, galaxy, etc.) can be considered from the point of view of celestial mechanics (the N-body problem with $N \gg 1$), fluid mechanics (the system is represented by a material continuum), or statistical mechanics (one defines a distribution function for the positions and the states of motion of the components of the system).

The three different approaches do not of course exclude one another, and very often they coexist in the treatment of certain problems. It may sound obvious if we state that the various problems are reduced and schematized in such a way that they can be looked at from one of the above viewpoints and with the tools that can be provided by the relevant discipline. However paradoxical it might appear (given the enormous amount of work produced by mathematicians on the N-body problem), it is our opinion that it is the first kind of approach which has received the least attention from the researchers on stellar dynamics or, at least, has received much less attention than the progress in the field could allow. If, from the publication of Chandrasekhar's book up to the present this has indeed happened it is due (in our opinion) mainly to two things.

The first is to do with the belief that the results of celestial mechanics always refer to only a few bodies and therefore cannot be applied to stellar dynamics (many bodies); in more concrete terms the situation has been such that the dialogue between those who have dealt with the problems of celestial mechanics (the mathematicians) and those who have dealt with the problems

[1] S. Chandrasekhar: *Principles of Stellar Dynamics* (University of Chicago Press, 1942; reprinted by Dover, New York, 1960), p. VII.

of stellar dynamics (the astrophysicists) has been minimal. We shall come back to this last point, later on.

The second and much more recent, concerns the ever-growing and somehow overwhelming use of computers. In the last decade, in particular, numerical simulations have a more and more important position with respect to analytical studies. Doubtless computers are an exceedingly powerful tool for investigating certain problems. However, in our opinion they should be used to single out those points on which to concentrate analytical study rather than as a short cut to avoid it. That is, computers should promote analytical study rather than replace it. This was the purpose of the well-known paper by Hénon and Heiles[2], which has opened up new horizons to a branch of mathematical physics. In addition, as proof of how essential the above-mentioned dialogue is, the paper deals with a stellar dynamical subject.

Furthermore, we are convinced that knowledge of the problems of the celestial mechanics (at a non-elementary level) is indispensable for those dealing with stellar dynamics: moreover, we think that there should not be a sharp boundary between the two disciplines. This led us to make the whole area the subject of a single book, albeit in two volumes. Our purpose is therefore to provide researchers in astronomy and astrophysics with an as thorough and clear an exposition as possible of the problems which constitute the foundations of celestial mechanics and stellar dynamics. It is therefore intended that for the latter the chosen approach is the first of those listed above.

There is now a general agreement that "mathematical" and "physical" cultures are quite distinct and that they also have difficulties, sometimes, in understanding each other. In our opinion, this situation, owing to the ever-increasing specialization of scientific learning, causes damage that is particularly severe in the field of astronomy and astrophysics.

Whereas in the past (we are speaking of a "golden age" that ended in the 1920s) the astronomer and the astrophysicist could take advantage of current work in mathematics and physics, today this is not only impossible but even unthinkable. The university education of astronomers and astrophysicists is overwhelmingly of the "physical" type: the mathematical tools acquired are inadequate for tackling the reading of any mathematical paper whatsoever (of mathematical physics, the theory of differential equations, etc.), which turns out to be necessary in some research. This is also because there is an irresistible tendency for everybody (and therefore mathematicians also) to retire into their own special language. Astronomers and astrophysicists, owing to the nature of the things they are dealing with, continually need to resort to results obtained by physicists and mathematicians. In the latter case, for the reason given above, that turns out to be exceedingly difficult and sometimes impossible. It is obvious, and we are convinced of this, that the times and cultural environments in which personalities like Poincaré,

[2] M. Hénon, C. Heiles: The applicability of the third integral of motion: some numerical experiments, *Astron. J.* **69**, 73–79 (1964).

Jeans or Eddington were present cannot come back again. However, we are also convinced that one can and must do something to better the present situation.

Our aim in planning, and in writing, this book has been to contribute to ferrying from the "mathematical" side to the "astronomical-astrophysical" side some of the results achieved in the last few decades, which we consider essential to anyone dealing with solar system, stellar systems, galactic dynamics, etc. It is clear that in an operation of this kind it may happen that some of the things to be ferried fall overboard, whether the boat was overloaded or the boatman not expert enough: we hope, however, to have kept the losses within acceptable limits. To continue with the methaphor, to the people living on the side at which the boat lands, we assume preparation to the intermediate graduate level (calculus, differential equations, vector calculus, ...).

We have done our best to provide a self consistent treatment, at least at a first level of understanding, to spare the reader continuous jumps from one textbook to another; at the same time we have also endeavoured to facilitate the deepening of individual arguments supplying indispensable information, including bibliographic details.

The point of view we have assumed is that of discussing the *problems* and not of going into the details of different applications: we have tried to single out the fundamental problems (i.e. the mathematical models) and to present them in as clear and readable a way as possible for a reader having the mathematical background assumed above. We also consider the reader to be fully acquainted with celestial mechanics at undergraduate level, to the extent that can be obtained, for instance, from an excellent book such as Danby's[3].

By tradition, the old textbooks on celestial mechanics used to include a chapter devoted to Hamiltonian mechanics, an indispensable tool for perturbation theory. We have not escaped from the tradition and the first volume includes a chapter devoted to selected topics of dynamics and dynamical systems. The second chapter, devoted to the two-body problem, is not meant to replace traditional expositions (which are assumed known to the reader) but simply to emphasize features of the problem which can prompt further developments. The third and the fourth chapter (the N-body problem and the three-body problem) follow on in the same spirit, giving much space to results so far to be found only in the original papers. The fifth chapter, to our mind, is intermediate between celestial mechanics and stellar dynamics as usually agreed upon. In all four chapters (from the second to the fifth), besides some novelties (we believe) in the planning of the material and the exposition of recent results, classical arguments sanctioned by tradition remain. For the latter, we have sometimes drawn our "inspiration" from the expositions of

[3] J. M. A. Danby: *Fundamentals of Celestial Mechanics*, 2nd Revised & Enlarged Edition (Willmann-Bell, Richmond, 1988).

authors whose works can now be considered "classics" and whom the reader will found mentioned in the notes to each chapter.

In the second volume the first three chapters are devoted to the theory of perturbations, starting from classical problems and arriving at the KAM theory and to the introduction of the use of the Lie transform. A whole chapter treats the theory of adiabatic invariants and its applications in celestial mechanics and stellar dynamics. Also the theory of resonances is illustrated and applications in both fields are shown. Classical and modern problems connected to periodic solutions are reviewed. The description of modern developments of the theory of Chaos in conservative systems is the subject of a chapter in which is given an introduction to what happens in both near-integrable and non-integrable systems. The invaluable help provided by computers in the exploration of the long time behaviour of dynamical systems is acknowledged in a final chapter where some numerical algorithms and their applications both to systems with few degrees of freedom and to large N-body systems, are illustrated.

A Note to the Reader

In this second volume, the formulae that appear in Vol. 1 are referred to by number, without the volume being specified. We hope this will not cause any misunderstanding since the numbering of the chapters is continuous throughout the two volumes. For the convenience of the reader, sometimes the formulae are rewritten; however, their original number is retained.

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Rome, September 1998

Dino Boccaletti
Giuseppe Pucacco

Contents

Contents of Volume 1