

Scattering from Model Nonspherical Particles

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Scattering from Model Nonspherical Particles

Theory and Applications
to Environmental Physics

With 95 Figures



Springer

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To my wife ... for reasons that are quite evident
to whomever knew her
Nando

Preface

The Mie theory is known to be the first approach to the electromagnetic scattering from homogeneous spheres endowed with all the accuracy of the Maxwell electromagnetic theory. It applies to spheres of arbitrary radius and refractive index and marks, therefore, noticeable progress over the approximate approach of Rayleigh, which applies to particles much smaller than the wavelength. As a consequence, after the publication of the Mie theory in 1908, several scattering objects, even when their shape was known to be nonspherical, were described in terms of equivalent spherical scatterers. It soon became evident, however, that the morphological details of the actual particles were often too important to be neglected, especially in some wavelength ranges. On the other hand, setting aside some particular cases in which the predictions of the Mie theory were acceptable, no viable alternative for the description of scattering from particles of arbitrary shape was at hand. This situation lasted, with no substantial changes, until about 25 years ago, when the exact solution to the problem of dependent scattering from aggregates of spheres was devised. This solution is a real improvement over the Mie theory because several processes that occur, e.g., in the atmospheric aerosols and in the interstellar medium, can be interpreted in terms of clustering of otherwise spherical scatterers. Moreover, nonspherical particles may be so distributed (both in size and orientation) as to smooth out the individual scattering properties. In this case, aggregates of spherical scatterers may be designed to approximate the real objects through a wise choice of the geometry and of the refractive indices of their components.

In the last 20 years, we have been active in the study of the scattering properties of aggregates of spheres, to whose theory we and our coworkers made some significant contributions that are, however, badly scattered throughout the literature. For this reason, we resolved to collect in this book these contributions organized within a coherent formalism, namely, the formalism of the multipole expansions in the framework of the transition matrix approach. The theory is supplemented by several results of calculations performed by ourselves and by our coworkers, whose inclusion is meant to yield information on the possible practical applications of the theory to systems of interest. In order to achieve uniformity of representation, the calculations were renewed and the results replotted. Therefore, this book should not

be understood as a treatise on scattering, although the general theory is described in some detail, but rather as a digest of the experience we have gained through our work in the field. The general topic of the comparison of our results with the experimental data or with the results yielded by other theoretical methods and approaches is pursued, but we do not pretend to be exhaustive because both the theoretical and the experimental study of scattering from actual nonspherical particles is a fast-moving field. Anyway, references to the relevant papers and books are given for all the cases of interest.

We start by summarizing in Chap. 1 the general theory of the spherical vector multipole fields, which are a basic tool for the development of the whole electromagnetic theory.

In Chap. 2 we recall the essentials of the theory of electromagnetic scattering and of the propagation of electromagnetic waves through dispersions of particles of arbitrary shape. The whole chapter is meant to establish a notation and to stress the need for introducing averages over the orientation of the particles in order to get predictions that are comparable to the observational data.

In Chap. 3 we expand the electromagnetic field in terms of spherical vector multipole fields and relate the expansion of the incident to that of the scattered field through the introduction of the transition matrix. The transformation properties of latter operator under rotation of the coordinate frame are exploited to show how orientational averages over an ensemble of particles can be performed. Explicit formulas for the averages of quantities of practical interest are given for a few instances of orientational distribution functions in which the averaging can be performed analytically.

In Chap. 4, after recalling the essentials of the Mie theory for homogeneous spheres and its extensions to layered and radially nonhomogeneous spheres, we introduce the equations that solve the problem for an aggregate of spherical scatterers of arbitrary geometry. The theory for homogeneous spheres containing one or more spherical inclusions is also expounded. In the last section of this chapter, we summarize the essentials of the discrete dipole approximation and of the finite difference time domain method. This summary is meant to help the reader to appreciate the advantages of the transition matrix approach and to better understand our considerations when comparing our results with those of other researchers in the field.

In Chap. 5 the topic of the scattering of particles in the presence both of a metallic and a dielectric plane interface is discussed. Even in this case the problem of the orientational average over the particles is dealt with in some detail.

The applications of the theory are described in Chap. 6, where the scattering properties of single and aggregated spheres and of spheres containing inclusions are considered, and in Chap. 7, where single and aggregated spheres and hemispheres in the presence of a plane interface are dealt with.

Finally, in Chaps. 8 and 9 we discuss how the theory can be applied in two specific contexts, the study of atmospheric ice crystals and of the interstellar dust grains, respectively.

We conclude this book with a short Appendix meant to review the mathematics implied in the calculations we presented. Actually, this appendix is rather cursory as we constantly resorted to mathematical tools that are quite standard in electromagnetics and in the theory of angular momentum. Our choice allowed us to use the mathematics that is reported in the existing literature instead of proving and reporting the needed significant relations. Anyway, in our opinion, the content of the mathematical appendix should be sufficient to give useful hints to whomever wants to face the problem of electromagnetic scattering through the transition matrix approach.

Before closing, we want to acknowledge the effort of our colleagues Dr. Orazio I. Sindoni, Prof. Giuditta Toscano, Dr. Enrico Fucile, Dr. Maria Antonia Iatì, Prof. Santi Aiello, and Dr. Cesare Cecchi-Pestellini who in time collaborated to the research on which this book is based. Finally, we want to express our sincere thanks to Prof. Michael I. Mishchenko for a critical reading of the manuscript and for several helpful suggestions and to Prof. Rodolfo Guzzi, whose advice and encouragement greatly helped us in writing this book.

Messina, Italy
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