

# *Engineering Materials*

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L. Ratke, P. W. Voorhees

# Growth and Coarsening

Ostwald Ripening  
in Material Processing

With 99 Figures



Springer

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TO SYLVIA AND MARIA

# Preface

Whenever a single phase system is quenched into a metastable state nucleation of a second phase lowers the free energy of the system. Following this nucleation event, in solid state transformations, the second phase typically grows by the diffusion of mass either into or from the matrix. When the composition of the matrix is nearly at its equilibrium value coarsening occurs. In this process the matrix is no longer a source or sink of solute, but the majority of the mass transport is from small to large particles. This process is driven by the decrease in interfacial energy per unit volume and results in an increase in the size scale of the two-phase mixture. The first to observe this coarsening process was Ostwald in 1900 who reported that the solubility of small HgO-particles is a function of the radius of the particle. The phenomenon has come to bear his name: Ostwald ripening. Ostwald ripening can occur in virtually any two-phase mixture in which there is significant diffusion. It is thus observed in two-phase mixtures ranging from liquid-vapor mixtures, such as droplets in clouds, to solid-liquid mixtures both during and following solidification. Understanding the dynamics of this ripening process and its surprising self-organizational nature has been at the center of the field since the seminal papers of Lifshitz and Slyozov, and Wagner appeared in the early 60's. From a more applied view, the Ostwald ripening process has a significant impact on manner in which materials are processed, e.g. age hardening of metallic alloys, grain refining, grain growth during recrystallisation, liquid and solid state sintering, and the deoxidation of steel melts. Given the ubiquitous nature of the ripening process and its clear technological importance, we feel that this subject deserves a unified development that is only possible in a monograph.

The main idea behind this monograph is to present students and researchers all the bits and pieces of coarsening theories such that they can understand the main issues and the underlying mathematics of self-similar coarsening of dispersed systems. Growth and coarsening are usually side notes in standard textbooks on physical metallurgy or materials science and as such they are treated in a very simplified fashion. Therefore, the monograph contains all of the background material necessary to understand growth and coarsening of spherical particles or droplets in a liquid or solid matrix. We do not refer the reader to any of the publications given in the bibliography for a derivation of a central result used in the monograph, but try to be self-consistent and comprehensive. Some basic knowledge of heat and mass transfer, thermodynamics, and differential equations would be helpful, but is not necessary as all the concepts required are presented, at least briefly. The monograph is suitable for advanced undergraduate students, graduate students, and researchers. The monograph is not a comprehensive review of coarsening and growth nor is it intended to give a complete survey of both topics. For example, we do not discuss the many mathematical approaches to the theory or the vast experimental literature. Rather, the monograph is intended to present a careful derivation of existing results and to place these results in some perspective.

Cologne and Evanston, July 2001

Lorenz Ratke and Peter W. Voorhees

# Table of Contents

<b>Preface</b> .....	V
<b>1. Introduction</b> .....	1
<b>2. Thermodynamics of alloys</b> .....	7
2.1 Basic definitions .....	7
2.2 Single component systems .....	11
2.3 Binary and multicomponent systems .....	15
2.4 Applications: Phase diagrams .....	28
2.4.1 Complete miscibility .....	28
2.4.2 Eutectic phase diagram .....	28
2.5 Gibbs-Thomson equation .....	31
2.5.1 Pure Material .....	32
2.6 Binary Alloy .....	35
<b>3. Transport of heat and mass</b> .....	43
3.1 Diffusive heat transport .....	43
3.2 Diffusive mass transport .....	45
3.3 Convective heat and mass transport .....	50
<b>4. Growth</b> .....	57
4.1 The general growth problem .....	57
4.2 Growth of a supercooled sphere .....	63
4.3 Growth in a supersaturated matrix .....	65
4.4 Interface Kinetics of first order .....	67
4.4.1 Interface kinetics in a supersaturated matrix ....	70
4.4.2 Interface kinetics in a supercooled melt .....	73
4.5 Interface kinetics of second order .....	74
4.5.1 Growth rates in second order kinetics .....	76
4.6 Convective contributions to heat and mass transfer ....	77



4.6.1	General growth laws at large Peclet numbers . . . .	78
4.6.2	Growth of a falling solid sphere . . . . .	80
4.6.3	Growth of a falling liquid sphere . . . . .	85
4.6.4	Growth at arbitrary Peclet numbers . . . . .	89
4.7	Growth with varying supersaturation . . . . .	92
<b>5.</b>	<b>Statistics of growth . . . . .</b>	<b>97</b>
5.1	Continuity equation and moments . . . . .	97
5.2	Statistics at constant growth rate . . . . .	99
5.3	Superposition of growth modes . . . . .	99
5.4	Constant supercooling or supersaturation . . . . .	100
5.4.1	Initial value problem - Cauchy problem . . . . .	101
5.4.2	Scaling solutions . . . . .	106
5.4.3	Scaling with the inverse square root of time . . . . .	109
5.4.4	Comments on scaling . . . . .	110
5.5	Separation ansatz solution for growth . . . . .	111
5.6	General solution for power law growth . . . . .	112
5.7	Convective diffusion to dispersed drops . . . . .	114
<b>6.</b>	<b>Coarsening - Basics and Growth Laws . . . . .</b>	<b>117</b>
6.1	General considerations on coarsening . . . . .	117
6.2	Ostwald ripening due to curvature supercooling . . . . .	119
6.3	Ostwald ripening due to solubility changes . . . . .	122
6.4	Ostwald ripening in interface kinetics or general first order reactions . . . . .	124
6.5	Coarsening due to a second-order reaction . . . . .	125
6.6	Coarsening under convective transport . . . . .	126
<b>7.</b>	<b>Ostwald ripening - Wagner analysis . . . . .</b>	<b>127</b>
7.1	Diffusion driven coarsening . . . . .	127
7.2	Coarsening with convective transport . . . . .	143
7.3	Second order reaction coarsening . . . . .	148
<b>8.</b>	<b>Ostwald ripening - Marqusee and Ross type analysis</b>	<b>151</b>
8.1	Interface driven coarsening in a supersaturated matrix .	151
8.2	Diffusive coarsening in a supercooled melt . . . . .	160
<b>9.</b>	<b>Multiparticle diffusion analysis . . . . .</b>	<b>167</b>
9.1	The Multiparticle Diffusion Problem . . . . .	168
9.2	Statistically averaged properties . . . . .	175

9.3	Effective Medium Theories .....	177
9.3.1	Averaging sphere models .....	177
9.3.2	Brailsford and Wynblatt approach .....	186
9.4	Statistical Mechanical Theories .....	191
9.5	Computer Simulations of Coarsening .....	195
<b>10.</b>	<b>Nucleation, Growth and Coarsening</b> .....	<b>205</b>
10.1	Nucleation and growth .....	206
10.1.1	General solution: nucleation and growth .....	210
10.1.2	Constant nucleation rate density .....	212
10.2	Nucleation, growth and coarsening .....	216
	<b>Appendix</b> .....	<b>227</b>
<b>A.</b>	<b>Droplet and particle motion</b> .....	<b>229</b>
A.1	Some basic fluid dynamic equations .....	229
A.2	Solid sphere moving in a gravity field .....	235
A.3	Droplet moving in a gravity field .....	241
A.4	Thermocapillary convection and motion .....	244
<b>B.</b>	<b>Nucleation</b> .....	<b>253</b>
B.1	Thermodynamics of nucleation .....	253
B.2	Kinetic theory of nucleation .....	261
B.2.1	Stationary nucleation rate .....	263
B.2.2	Time dependent nucleation .....	267
B.3	Heterogeneous nucleation .....	268
	<b>Bibliography</b> .....	<b>271</b>
	<b>List of Variables</b> .....	<b>285</b>
	<b>Index</b> .....	<b>295</b>